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METHOD OF EDGE WAVES IN THE PHYSICAL
THEORY OF DIFFRACTION

by

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EDITED TRANSLATION

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The book is a monograph written as a result of research by the author. The diffraction of plane electromagnetic waves by ideally conducting bodies, the surface of which have discontinuities, is investigated in the book. The linear dimensions of the bodies are assumed to be large in comparison with the wavelength. The method developed in the book takes into account the perturbation of the field in the vicinity of the surface discontinuity and allows one to substantially refine the approximations of geometric and physical optics. Expressions are found for the fringing field in the distant zone. A numerical calculation is performed of the scattering characteristics, and a comparison is made with the results of rigorous theory and with experiments.

The book is intended for physicists and radio engineers who are interested in diffraction phenomena, and also for students of advanced courses and aspirants who are specializing in antennas and the propagation of radio waves.

FOREWORD

First of all, one should explain the term "physical theory of diffraction". In order to do this, let us discuss briefly the historical development of diffraction theory.

If one investigates, for example, the incidence of a plane electromagnetic wave on a body which conducts well, all the dimensions of which are large in comparison with the wavelength, then the simplest solution of this problem may be obtained by means of geometric optics. It is known that in a number of cases one must add to geometric optics the laws of physical optics which are connected with the names of Huygens, Fresnel, Kirchhoff and Cotler. Physical optics uses, together with the field equations, the assumption that in the vicinity of a reflecting body geometric optics is valid.

At the start of the Twentieth Century, a new division of mathematical physics appeared — the mathematical theory of diffraction. Using it, rigorous solutions to the problem of diffraction by a wedge, sphere, and infinite cylinder were obtained. Subsequently, other rigorous solutions were added; however, the total number of solutions was relatively small. For sufficiently short waves (in comparison with the dimensions of the body or other characteristic distances) these solutions, as a rule, are ineffective. Here the direct numerical methods also are unsuitable.

Hence, an interest arose in approximation (asymptotic) methods which would allow one to investigate the diffraction of sufficiently short waves by various bodies, and would lead to more precise and reliable quantitative results than does geometric or physical optics. Obviously, these methods must in some way be considered the most important results extracted from the mathematical theory of diffraction.

In the "geometric theory of diffraction" proposed by Keller, the results obtained in the mathematical theory of diffraction of short waves were exactly the ones which were used and generalized. Here, the concept of diffraction rays advanced to the forefront. This concept was expressed rather as a physical hypothesis and was not suitable for representing the field in all of space: it was not usable where the formation of the diffraction field takes place (at the caustic, at the boundary of light and shadow, etc.). Here it is impossible to talk about rays, and one must use a wave interpretation.

What has been said above makes it clear why a large number of works appeared in which the diffraction of short waves was investigated by other methods. Among those applied to reflecting bodies with abrupt surface discontinuities or with sharp edges (strip, disk, finite cylinder or cone, etc.) one should first of all mention the works of P. Ya. Ufimtsev. These works began to appear in print in 1957, and it is on the basis of them that this book was written.

P. Ya. Ufimtsev studied the scattering characteristics by such bodies by taking into account, besides the currents being excited on the surface of the body according to the laws of geometric optics (the "uniform part of the current" according to his terminology), the additional currents arising in the vicinity of the edges or borders which have the character of edge waves and rapidly attenuate with increasing distance from the edge or border (the "nonuniform part of the current"). One may find the radiation field created by the additional currents by comparing the edge or border with the edge of an infinite wedge or the border of a half-plane. In certain cases, one is obliged to consider the diffraction interaction of the various edges — that is, the fact that the wave created by one edge and propagated past another edge is diffracted by it (secondary diffraction).

Such an approach to the diffraction of short waves has great physical visualizability and allows one to obtain rather simple approximation expressions for the field scattered by various metal bodies. This approach may be called the <u>physical theory of diffraction</u>

This name is applied to many works on the diffraction of short waves in which the mathematical difficulties are bypassed by means of physical considerations.

It is clear that the physical theory of diffraction is a step forward in comparison with physical optics, which in general neglects the additional (edge) currents. The results obtained in this book show that with a given wavelength the physical theory of diffraction gives a better precision than physical optics, and with a given precision the physical theory of diffraction allows one to advance into the longer wave region and, in particular, to obtain a number of results which are of interest for radar where the ratios of the dimensions of the bodies to the wavelength do not reach such large values as in optics.

In addition, the physical theory of diffraction encompasses a number of interesting phenomena which are entirely foreign to physical optics. Thus, in a number of cases the additional currents give, not a small correction to the radiation field, but the main contribution to this field (see especially Chapters IV and V). If a plane wave is diffracted by a thin straight wire (a passive vibrator), then the additional current falls off very slowly as one goes further from the end of the wire. Therefore, the solution is obtained by summing the entire array of diffraction waves (secondary, tertiary, etc.) which successively arise as a consequence of the reflection of the currents from the ends of the wires. It has a resonance character. Thus, the problem of the scattering of the plane wave by a finite length wire which is a diffraction problem of a slightly unusual type is solved in Chapter VII. The resulting solution is applicable under the condition that the diameter of the wire is small in comparison with the wavelength and length of the wire, and the ratio of the length of the wire to the wavelength is arbitrary.

The final equations which are derived in this book and are used for calculations are not asymptotic in the strict sense of the word.

Therefore, it is natural to pose the question: in what way will the subsequent asymptotic equations differ from them when at last one obtains them in the mathematical theory of diffraction? One can say beforehand that the main term of the asymptotic expansion will not, in the general case, agree with the solution obtained on the basis of physical considerations: other (as a rule more complicated) slowly varying functions which determine the decay of the fields and currents as one goes further from the edges and borders, and also the diffraction interaction of the edges and the shadowing of the edge waves will figure in the main term. However, the refinement of the slowly varying functions in the expression for the diffraction field is not able to seriously influence the quantitative relationships. This is seen from a comparison of the results obtained in this book with calculations based on rigorous theory and other approximation equations, and also with the results of measurements.

The relationships obtained in this book also should help the development of asymptotic methods in the mathematical theory of diffraction, since they suggest the character of the approximations and the structure of the desired solution.

L. A. Vaynshteyn

INTRODUCTION

In recent years, there has been a noticeable increase of interest in the diffraction of electromagnetic waves by metal bodies of complex shape. Such diffraction problems with a rigorous methematical formulation reduce to an interpretation of the wave equation or Maxwell equations with consideration of the boundary conditions on the body's surface. However, one cannot succeed in finding solutions in the case of actual bodies of a complicated configuration. This may be done only for bodies of the simplest geometric shape — such as an infinitely long cylinder, a sphere, a disk, etc. It turns out that the resulting solutions permit one to effectively calculate the diffraction field only under the condition that the wavelength is larger than, or comparable to, the finite dimensions of the body. In the "quasi-optical"case, when the wavelength is a great deal less than the dimensions of the body, the rigorous solutions usually lose their practical value, and it is necessary to add to them laborious and complicated asymptotic studies. Here, the numerical methods for the solution of boundary value problems also become ineffective. Therefore, in the theory of diffraction the approximation methods which allow one to study the diffraction of sufficiently short waves by various bodies acquire great importance.

The field scattered by a given body may be calculated approximately by means of geometric optics laws (the reflection equations, see,for example [1-3]), from the principles of Huygens-Fresnel and from the equations of Kirchhoff and Cotler [3-6].

The most common method of calculation in the quasi-optic case is the principle of Huygens-Fresnel in the formulation of Kirchhoff and Cotler — the so-called physical optics approach. The essence of this method may be summarized as follows.

Let a plane electromagnetic wave fall on some ideally conducting body which is found in free space. In the physical optics approach, the surface current density which is induced by this wave on the irradiated part of the body's surface is (in the absolute system of units) equal to

$$\mathbf{j}^0 = \frac{c}{2\pi}[\mathbf{n}\mathbf{H}_0], \qquad (A)$$

where c is the speed of light in a vacuum, $\mathbf{n}$ is the external normal to the body's surface, H_0 is the magnetic field of the incident wave. On the darkened side of the body the surface current is assumed to be equal to zero ($j^0 = 0$). Equation (A) means that on each element of the body's irradiated surface the same current is excited as on an ideally conducting surface of infinite dimensions tangent to this element. The scattered field created by the current (A) is then found by means of Maxwell's equations.

It is obvious that in reality the current induced on the body's surface will differ (as a consequence of the curve of the surface) from the current j^0. The precise expression for the surface current density has the form

$$\mathbf{j} = \mathbf{j}^0 + \mathbf{j}^1, \qquad (B)$$

where j^1 is the surface density of the additional current which results from the curve of the surface. By the curve of the surface, we mean any of its deviations from an infinite plane (a smooth curve, a sharp bend, a bulge, a hole, etc.). If the body is convex and smooth and its dimensions and radii of curvature are large in comparison with the wavelength, then the additional current is concentrated mainly in the vicinity of the boundary between the illuminated and shadowed parts of the body's surface. But if the body has an edge, bend, or point, then the additional current also arises near them. The additional current density is comparable to the density j^0, as a rule, only at distances of the order of a wavelength from the corresponding edge, bend, or point. Thus, if the body's dimensions significantly exceed the wavelength, the additional currents occupy a comparatively small part of its surface.

Since the current excited by the plane wave on an ideally conducting surface is distributed uniformly over it (the absolute magnitude of its surface density is constant) then the vector j^0 may be called the "uniform" part of the surface current. The additional current j^1 which is caused by the curve of the body's surface we will henceforth call the "nonuniform" part of the current. In the physical optics approach, only the uniform part of the current is considered. Therefore, it is no wonder that in a number of cases it gives unsatisfactory results. For a more precise calculation, it is necessary to also take into account the nonuniform part of the current.

In this book, the results of the author relating to the approximation solution of diffraction problems are discussed and systematized. Essentially, these results were briefly discussed in a number of papers [7-14]. Roughly at the same time, the works of other authors devoted to similar problems appeared. We will discuss them in more detail (in §25) after the reader becomes accustomed to the concepts being used in diffraction problems of this type. For the present, let us only note that in these works, as a rule, other methods are used.

In the book, problems of the diffraction of plane electromagnetic waves by complex metal bodies, the surfaces of which have discontinuities (edges), are investigated. The dimensions of the bodies are assumed to be large in comparison with the wavelength, and their surface is assumed to be ideally conducting.

Obviously, if the edges are sufficiently far from one another, then the current flowing on a small element of the body's surface in the vicinity of its discontinuity may be approximately considered to be the same as on a corresponding infinite dihedral angle (a wedge). In fact, in Chapter I it is shown (see also [5] §20) that the nonuniform part of the current on a wedge has the character of an edge wave which rapidly decreases with the distance from the edge. Therefore, one may consider that the nonuniform part of the current is concentrated mainly in the vicinity of the discontinuity. By means of this physically obvious assumption, the field scattered by a strip (Chapter I), by a disk (Chapter II), by a finite length cylinder (Chapter III) and by certain other bodies of rotation (Chapter IV) is calculated.

For a more precise calculation, however, it is necessary to keep in mind that the actual current distribution in the vicinity of the body's edges differs from the current distribution near the edge of the wedge. Actually, the edge wave corresponding to the nonuniform part of the current, propagated along the body's surface, reaches the adjacent edge and undergoes diffraction by it, exciting secondary edge waves. The latter in turn produce new edge waves, etc. If all the dimensions of the body are large in comparison with the wavelength, then as a rule it is sufficient to consider only the secondary diffraction. This phenomenon is studied in Chapter V using the example of a strip and disk.

In the case of a narrow cylindrical conductor of finite length, the edge waves of the current decrease very slowly with the distance from each end. Therefore, here it is impossible to limit oneself to a consideration only of secondary diffraction, and it is necessary to investigate the multiple diffraction of edge waves. Chapter VII is devoted to this problem.

The uniform and nonuniform parts of the current are more than auxiliary concepts which are useful in solving diffraction problems. In Chapter VI it is shown that one is able experimentally to separate from the total fringing field that part of it which is created by the nonuniform part of the current. There, it is also shown that the depolarization phenomenon of the reflected signal is caused only by the nonuniform part of the current.

Let us note the following feature of the method discussed in the book. A physical representation of the nonuniform part of the current is widely used in the book, but nowhere are its explicit mathematical expressions cited. This part of the current is generally not expressed in terms of well-known functions. Obviously a direct integration of the currents when calculating the fringing field is able to lead only to very complicated and immense equations. Therefore, we will find the fringing field created by the nonuniform part of the current on the basis of indirect considerations without direct integration of it (see especially Chapters I - IV).

The method by which the diffraction problems are solved in this book may be briefly summarized as follows. We will seek an approximate solution of the diffraction problem for a certain body by first having studied diffraction by its separate geometric elements. For example, for a finite cylinder such elements are: the lateral surface as part of an infinite cylindrical surface, each base as part of a plane, each section of the base rim as the edge of a wedge (the curvature of the rim in the first approximation may be neglected). Having studied the diffraction by the separate elements of the body, we will obtain a representation of the nonuniform part of the current and of the field which is radiated by it. Then secondary, tertiary, etc. diffraction is studied — that is, the diffraction interaction of the various elements of the body is taken into account.

This method appeals to physical considerations, not only when formulating the problem but also in its solution process, and in this way differs from the methods of the mathematical theory of diffraction. Therefore, such a method may be referred to as the <u>physical</u> theory of diffraction.

A whole series of other diffraction studies which appeared in the last five to ten years also are able to relate to the physical theory of diffraction. The first work which contained the idea of the physical theory of diffraction is evidently the paper of Schwarzschild [15] which was published at the beginning of this century and was devoted to diffraction by a slit.

One should note that approximate solutions of diffraction problems would be impossible without the use of the results obtained in the mathematical theory of diffraction. In particular, the rigorous solution to the problem of diffraction by a wedge which is attributed to Sommerfeld [16] is widely used in this book. In Chapter I this solution is obtained by another method. The works of Fok [17, 18] served as the starting point for numerous studies on diffraction by smooth convex bodies. The rigorous solution of the problem of diffraction at the open end of a wave guide [19] revealed the mechanism for the formation of primary diffraction waves, and their shadowing by the opposite end of the wave guide. The rigorous theory as applied to a strip and disk allows us to examine the precision of the approximation theory (see Chapter V).

CHAPTER I

DIFFRACTION BY A WEDGE

As was already said in the Introduction, the field scattered by a body may be investigated in the form of the sum of the fields being radiated by the uniform and nonuniform parts of the surface current. The uniform part of the current is completely determined by the geometry of the body and the magnetic field of the incident wave. The nonuniform part generally is unknown. However, one may approximately assume that in the vicinity of the discontinuity of a convex surface it will be the same as on a corresponding wedge. Therefore, it is necessary for us to begin by studying the diffraction of a plane electromagnetic wave by a wedge. This chapter will be devoted to this problem. First we will investigate the rigorous solution of this problem (§ 1 and 2). Then we will find its solution in the physical optics approach (§ 3). The difference of these solutions determines the field created by the nonuniform part of the current (§ 4).

§ 1. The Rigorous Solution

The rigorous solution to the problem of diffraction of a plane wave by a wedge was first obtained by Sommerfeld by the method of branching wave functions [16]. Later, the diffraction of cylindrical and spherical waves by a wedge also was studied. A rather extensive bibliography on these problems may be found, for example, in the paper of Oberhettinger [20]. Since the problem of diffraction by a wedge lies at the base of our studies, we considered it advisable not only to present the results of its rigorous solution, but also to give them a new more graphic derivation. The idea for this derivation follows directly from the work of Sommerfeld. Sommerfeld found the solution to the problem in the form of a contour integral, and then he transformed it to a series. However, one may proceed in the opposite direction: first find the solution in the form of a series and then give its integral representation. Such a path seems to us more graphic, and is discussed in this section. The necessity for a detailed derivation is caused by the fact that the results of Sommerfeld [16] are not represented in a sufficiently clear form, which hinders their use.

Let us assume there is in free space (a vacuum) an ideally conducting wedge and a cylindrical wave source Q parallel to its edge (Figure 1). Let us introduce the cylindrical coordinate system r, ϕ, z in such a way that the z axis coincides with the wedge edge, and the angle ϕ is measured from the irradiated surface. The external wedge angle will be designated by the letter α, so that $0 \leqslant \varphi \leqslant \alpha$. The coordinates of the source Q we will designate by r_0, ϕ_0.

Let us investigate two particular cases for the excitation of an electromagnetic field. In the first case, it is excited by a "filament of electric current"

$$j_z^e = -i\omega p_z \delta(r - r_0, \varphi - \varphi_0), \tag{1.01}$$

in the second case, it is excited by a "filament of magnetic current"

$$j_z^m = -i\omega m_z \delta(r - r_0, \varphi - \varphi_0). \tag{1.02}$$

The quantities p_z and m_z here designate, respectively, the electric and magnetic moments of the filament per unit length along the z axis, ω is the cyclic frequency $\left(\omega = ck = c\frac{2\pi}{\lambda}\right)$, $\delta(r - r_0, \varphi - \varphi_0) = \delta(r - r_0)\,\delta[r(\varphi - \varphi_0)]$ is a two-dimensional delta function which satisfies the condition

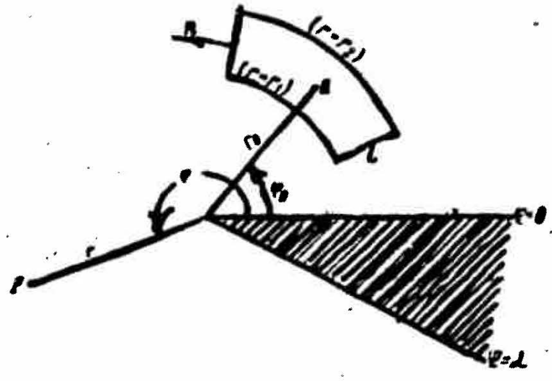


Figure 1. The excitation of a wedge-shaped region by a linear source.

Q - source; P - the observation point;
L - the integration contour in Equation (1.10).

$$\iint \delta(r - r_0, \varphi - \varphi_0)\, r dr\, d\varphi = 1$$

with integration over the neighborhood of the point r_0, ϕ_0.

Here and henceforth, we will use the absolute system of units (the Gauss system), and we will assume the dependence on time is in the form $e^{-i\omega t}$.

In the first case, the "electric" vector potential A_z^e satisfies the equation (see, for example, [4])

$$\Delta A_z^e + k^2 A_z^e = -\frac{4\pi}{c} j_z^e \qquad (1.03)$$

and the boundary condition

$$A_z^e = 0 \quad \text{with} \quad \varphi = 0 \text{ and } \varphi = \alpha. \qquad (1.04)$$

In the second case, the "magnetic" vector potential A_z^m satisfies the equation

$$\Delta A_z^m + k^2 A_z^m = -\frac{4\pi}{c} j_z^m \tag{1.05}$$

and the boundary condition

$$\frac{\partial A_z^m}{\partial \varphi} = 0 \quad \text{with} \quad \varphi = 0 \ \text{and} \ \varphi = \alpha. \tag{1.06}$$

It is natural to seek the solution of the nonhomogeneous Equations (1.03) and (1.05) in the form

$$A_z^e = \begin{cases} \sum_{s=1}^{\infty} a_s J_{\nu_s}(kr) H_{\nu_s}^{(1)}(kr_0) \sin \nu_s \varphi_0 \sin \nu_s \varphi \ \text{with} \ r < r_0, \\ \sum_{s=1}^{\infty} a_s J_{\nu_s}(kr_0) H_{\nu_s}^{(1)}(kr) \sin \nu_s \varphi_0 \sin \nu_s \varphi \ \text{with} \ r > r_0; \end{cases} \tag{1.07}$$

$$A_z^m = \begin{cases} \sum_{s=0}^{\infty} b_s J_{\nu_s}(kr) H_{\nu_s}^{(1)}(kr_0) \cos \nu_s \varphi_0 \cos \nu_s \varphi \ \text{with} \ r < r_0, \\ \sum_{s=0}^{\infty} b_s J_{\nu_s}(kr_0) H_{\nu_s}^{(1)}(kr) \cos \nu_s \varphi_0 \cos \nu_s \varphi \ \text{with} \ r > r_0, \end{cases} \tag{1.08}$$

$$\nu_s = s\frac{\pi}{\alpha}.$$

The products

$$\begin{matrix} J_{\nu_s}(kr) \sin \nu_s \varphi & & J_{\nu_s}(kr) \cos \nu_s \varphi \\ & \text{and} & \\ H_{\nu_s}^{(1)}(kr) \sin \nu_s \varphi & & H_{\nu_s}^{(1)}(kr) \cos \nu_s \varphi \end{matrix} \tag{1.09}$$

are the partial solutions of Equations (1.03) and (1.05) without the right-hand member which satisfy the boundary conditions (1.04) and (1.06). The remaining factors entering into Equations (1.07) and (1.08) ensure the observance of the reciprocity principle and the continuity of the field on the arc $r = r_0$. The Bessel function $J_{\nu_s}(kr)$ enters these equations when $r < r_0$, because it remains finite when $r \to 0$, and the Hankel function $H_{\nu_s}^{(1)}(kr)$ is taken when $r > r_0$ in order that the solution satisfies the radiation condition.

The coefficients a_s and b_s may be determined by means of Green's theorem

$$\oint_L \frac{\partial u}{\partial n} dl = \int \Delta u dS, \quad dS = r dr d\varphi \tag{1.10}$$

for the contour L in the plane z = const which is shown in Figure 1. Here, the external normal to the contour L is designated by the letter n. Applying Equation (1.10) to the functions A_z^e and A_z^m and performing the limiting transitions $r_1 \to r_0$ and $r_2 \to r_0$ in it, we obtain

$$\int \left(\left.\frac{\partial A_z^e}{\partial r}\right|_{r_0+0} - \left.\frac{\partial A_z^e}{\partial r}\right|_{r_0-0} \right) r_0 d\varphi = i \frac{4\pi k}{r_0} p_z \int \delta(\varphi - \varphi_0) r_0 d\varphi,$$
$$\int \left(\left.\frac{\partial A_z^m}{\partial r}\right|_{r_0+0} - \left.\frac{\partial A_z^m}{\partial r}\right|_{r_0-0} \right) r_0 d\varphi = i \frac{4\pi k}{r_0} m_z \int \delta(\varphi - \varphi_0) r_0 d\varphi.$$

Since here the integration limits are arbitrary, it follows from the equality of the integrals that the integrands are equal:

$$\left.\frac{\partial A_z^e}{\partial r}\right|_{r_0+0} - \left.\frac{\partial A_z^e}{\partial r}\right|_{r_0-0} = i \frac{4\pi k p_z}{r_0} \delta(\varphi - \varphi_0), \tag{1.11}$$

$$\left.\frac{\partial A_z^m}{\partial r}\right|_{r_0+0} - \left.\frac{\partial A_z^m}{\partial r}\right|_{r_0-0} = i \frac{4\pi k m_z}{r_0} \delta(\varphi - \varphi_0). \tag{1.12}$$

Now let us substitute Expressions (1.07) into Equality (1.11) and multiply both members of the latter by $\sin \nu_s \varphi$. Then integrating the resulting equality over ϕ in the limits from 0 to α, we find

$$a_s = \frac{4\pi^2}{\alpha} k p_z. \tag{1.13}$$

In a similar way, let us determine the coefficients

$$b_s = \varepsilon_s \frac{4\pi^2}{\alpha} k m_z, \tag{1.14}$$

where

$$\varepsilon_0 = \frac{1}{2}, \quad \varepsilon_1 = \varepsilon_2 = \ldots = 1. \tag{1.15}$$

Consequently, the electric current filament excites, in the space outside the wedge, the field

$$E_z = ikA^e_z =$$

$$= \begin{cases} i\dfrac{4\pi^2}{\alpha} k^2 p_z \sum\limits_{s=1}^{\infty} H^{(1)}_{\nu_s}(kr_0)\, J_{\nu_s}(kr) \sin\nu_s\varphi_0 \sin\nu_s\varphi \\ \qquad\qquad\qquad\qquad \text{with } r < r_0, \\ i\dfrac{4\pi^2}{\alpha} k^2 p_z \sum\limits_{s=1}^{\infty} J_{\nu_s}(kr_0)\, H^{(1)}_{\nu_s}(kr) \sin\nu_s\varphi_0 \sin\nu_s\varphi \\ \qquad\qquad\qquad\qquad \text{with } r > r_0, \end{cases}$$

$$E_r = E_\varphi = 0, \quad \mathbf{H} = \frac{1}{ik}\,\mathrm{rot}\,\mathbf{E}, \tag{1.16}$$

and the magnetic current filament excites, outside the wedge, the field

$$H_z = ikA^m_z =$$

$$= \begin{cases} i\dfrac{4\pi^2}{\alpha} k^2 m_z \sum\limits_{s=0}^{\infty} \varepsilon_s H^{(1)}_{\nu_s}(kr_0) J_{\nu_s}(kr) \cos\nu_s\varphi_0 \cos\nu_s\varphi \\ \qquad\qquad\qquad\qquad \text{with } r < r_0, \\ i\dfrac{4\pi^2}{\alpha} k^2 m_z \sum\limits_{s=0}^{\infty} \varepsilon_s J_{\nu_s}(kr_0)\, H^{(1)}_{\nu_s}(kr) \cos\nu_s\varphi_0 \cos\nu_s\varphi \\ \qquad\qquad\qquad\qquad \text{with } r > r_0, \end{cases}$$

$$H_r = H_\varphi = 0, \quad \mathbf{E} = -\frac{1}{ik}\,\mathrm{rot}\,\mathbf{H}. \tag{1.17}$$

Now using the asymptotic equation for the Hankel function when [21], we have

$$H^{(1)}_{\nu_s}(kr_0) = \sqrt{\frac{2}{\pi k r_0}}\, e^{i\left(kr_0 - \frac{\pi}{2}\nu_s - \frac{\pi}{4}\right)} = H^{(1)}_0(kr_0)\, e^{-i\frac{\pi}{2}\nu_s}. \tag{1.18}$$

Then Expressions (1.16) and (1.17) in the region $r < r_0$ take the form

$$E_z = i\frac{4\pi^2}{\alpha} k^2 p_z H^{(1)}_0(kr_0) \times$$

$$\times \sum_{s=1}^{\infty} e^{-i\frac{\pi}{2}\nu_s} J_{\nu_s}(kr) \sin\nu_s\varphi_0 \sin\nu_s\varphi,$$

(equation continued on next page)

$$H_z = i\frac{4\pi^2}{\alpha}k^2 m_z H_0^{(1)}(kr_0)\times$$
$$\times\sum_{s=0}^{\infty}\varepsilon_s e^{-i\frac{\pi}{2}\nu_s}J_{\nu_s}(kr)\cos\nu_s\varphi_0\cos\nu_s\varphi$$

or

$$\left.\begin{aligned} E_z &= i\pi k^2 p_z H_0^{(1)}(kr_0)\left[u(r,\ \varphi-\varphi_0)-u(r,\ \varphi+\varphi_0)\right],\\ H_z &= i\pi k^2 m_z H_0^{(1)}(kr_0)\left[u(r,\ \varphi-\varphi_0)+u(r,\ \varphi+\varphi_0)\right],\end{aligned}\right\} \qquad (1.19)$$

where

$$u(r,\ \psi)=\frac{2\pi}{\alpha}\sum_{s=0}^{\infty}\varepsilon_s e^{-i\frac{\pi}{2}\nu_s}J_{\nu_s}(kr)\cos\nu_s\psi \qquad (1.20)$$

$(\psi=\varphi\pm\varphi_0)$.

Let us note, furthermore, that in free space the field of the electric filament with a moment p_z is determined by the relationship

$$E_z = i\pi k^2 p_z H_0^{(1)}(kr_0), \qquad (1.21)$$

and the field of the magnetic filament with the moment m_z is determined by the relationship

$$H_z = i\pi k^2 m_z H_0^{(1)}(kr_0). \qquad (1.22)$$

Therefore, the expressions in front of the square brackets in Equations (1.19) may be regarded as the primary field of the filament — the cylindrical wave arriving at the wedge edge. Now removing the filament of current to infinity ($r_0 \to \infty$), let us proceed to the incident plane waves

$$E_z = E_{0z}e^{-ikr\cos(\varphi-\varphi_0)},\quad E_r = E_\varphi = 0 \qquad (1.23)$$

and

$$H_z = H_{0z} \cdot e^{-ikr\cos(\varphi-\varphi_0)}, \quad H_r = H_\varphi = 0. \tag{1.24}$$

The field arising with the diffraction of these waves by the wedge will obviously have the component

$$E_z = E_{0z}[u(r, \varphi-\varphi_0) - u(r, \varphi+\varphi_0)] \tag{1.25}$$

and

$$H_z = H_{0z}[u(r, \varphi-\varphi_0) + u(r, \varphi+\varphi_0)]. \tag{1.26}$$

Let us find the integral representation for the function u (r, ψ). For this purpose, let us use the equation (see [16], p. 866)

$$J_{\nu_s}(kr) = \frac{1}{2\pi}\int_I^{III} e^{i\left[kr\cos\beta + \nu_s\left(\beta - \frac{\pi}{2}\right)\right]} d\beta, \tag{1.27}$$

where the limits I - III mean that the integration contour goes from region I to region III (Figure 2). The cross-hatched sections in the plane of the complex variable β (β') shown in Figure 2 are regions in which $\operatorname{Im}\cos\beta > 0$ ($\operatorname{Im}\cos\beta' < 0$). Therefore, in the sections of the contour extending to infinity the integrand strives to zero, ensuring the convergence of the integral. Substituting Expression (1.27) into Equation (1.20), we obtain

$$u(r, \psi) = \\ = \frac{1}{2\alpha}\int_I^{III} e^{ikr\cos\beta}\left[1 + \sum_{s=1}^{\infty} e^{i\nu_s(\beta-\pi+\psi)} + \sum_{s=1}^{\infty} e^{i\nu_s(\beta-\pi-\psi)}\right] d\beta.$$

After summing the infinite geometric progressions and replacing the variable β by $\beta' = \beta - \pi$, the function u (r, ψ) acquires the form

$$u(r, \psi) = \\ = \frac{1}{2\alpha}\int_{I'}^{III'} e^{-ikr\cos\beta'}\left[\frac{1}{1 - e^{i\frac{\pi}{\alpha}(\beta'+\psi)}} - \frac{1}{1 - e^{-i\frac{\pi}{\alpha}(\beta'-\psi)}}\right] d\beta'.$$

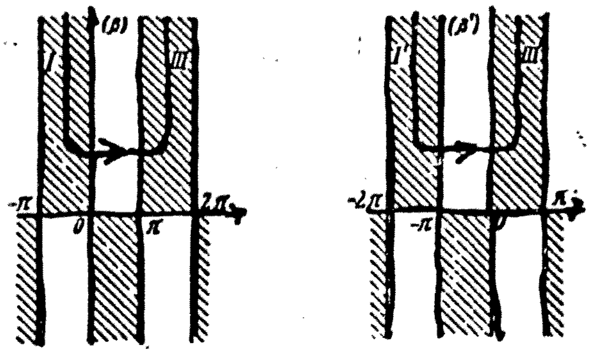


Figure 2. The integration contours in the complex plane β.

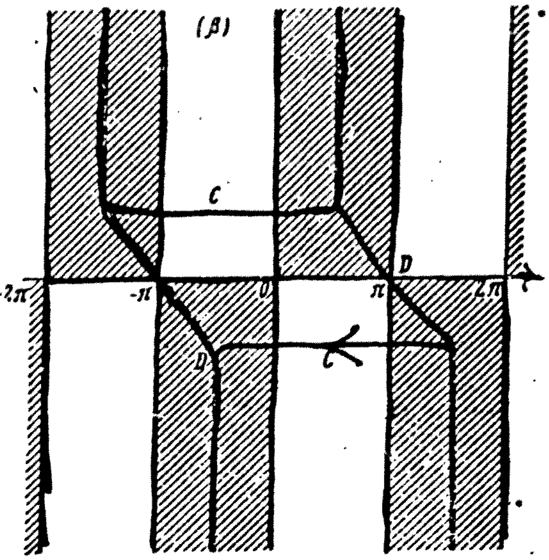


Figure 3. The integration contour in Equation (1.28).

As a result, we obtain the well-known integral of Sommerfeld

$$u(r, \psi)=\frac{1}{2\alpha}\int\limits_{C}\frac{e^{-ikr\cos\beta}}{1-e^{i\frac{\pi}{\alpha}(\beta+\phi)}}\,d\beta. \tag{1.28}$$

The integration contour C is shown in Figure 3 and consists of two infinite branches. Since the integrand expression has poles at the points $\beta_m=2\alpha m-\phi\ (m=0, \pm 1, \pm 2, \ldots)$, then for the values of ϕ corresponding to the space outside the wedge $(0<\varphi<\alpha)$ the function u (r, ψ) may be represented (with $\pi<\alpha\leqslant 2\pi,\ 0\leqslant\varphi_0\leqslant\pi$) in the following way:

$$\left.\begin{array}{ll} u(r, \psi)=v(r, \psi)+e^{-ikr\cos\phi} & \text{with } -\pi<\psi<\pi, \\ u(r, \psi)=v(r, \psi) & \text{with } \pi<\psi<2\alpha-\pi, \\ u(r, \psi)=v(r, \psi)+e^{-ikr\cos(2\alpha-\phi)} & \\ & \text{with } 2\alpha-\pi<\psi\leqslant 2\alpha, \end{array}\right\} \tag{1.29}$$

where

$$v(r, \psi)=\frac{1}{2\alpha}\int\limits_{B}\frac{e^{-ikr\cos\beta}}{1-e^{i\frac{\pi}{\alpha}(\beta+\phi)}}\,d\beta,$$

or

$$v(r, \psi)=\frac{i}{2\alpha}\sin\frac{\pi^2}{\alpha}\int\limits_{D_0}\frac{e^{ikr\cos\zeta}d\zeta}{\cos\frac{\pi^2}{\alpha}-\cos\frac{\pi}{\alpha}(\psi+\zeta)}. \tag{1.30}$$

The integration contours D and D_0 are shown, respectively, in Figures 3 and 4.

With an arbitrary incidence of a plane wave on a wedge, one of two cases may occur: (1) the plane wave "illuminates" only one face of the wedge $(0<\varphi_0<\alpha-\pi)$, and (2) the plane wave "illuminates" both faces of the wedge $(\alpha-\pi<\varphi_0<\pi)$. Let us write out in more detail the functions u (r, ψ) corresponding to these cases. In the case $\varphi_0<\alpha-\pi$ (Figure 5), we have

$$\left.\begin{array}{ll}\left.\begin{array}{l}u(r, \varphi-\varphi_0)=\\=v(r, \varphi-\varphi_0)+e^{-ikr\cos(\varphi-\varphi_0)}\\u(r, \varphi+\varphi_0)=\\=v(r, \varphi+\varphi_0)+e^{-ikr\cos(\varphi+\varphi_0)}\end{array}\right\}&\text{with}\ 0<\varphi<\pi-\varphi_0,\\\left.\begin{array}{l}u(r, \varphi-\varphi_0)=\\=v(r, \varphi-\varphi_0)+e^{-ikr\cos(\varphi-\varphi_0)}\\u(r, \varphi+\varphi_0)=v(r, \varphi+\varphi_0)\end{array}\right\}&\text{with}\ \pi-\varphi_0<\varphi<\pi+\varphi_0,\\\left.\begin{array}{l}u(r, \varphi-\varphi_0)=v(r, \varphi-\varphi_0)\\u(r, \varphi+\varphi_0)=v(r, \varphi+\varphi_0)\end{array}\right\}&\text{with}\ \pi+\varphi_0<\varphi\leqslant\alpha,\end{array}\right| \tag{1.31}$$

and in the case $\alpha-\pi<\varphi_0<\pi$ (Figure 6) we have

$$\left.\begin{array}{ll}\left.\begin{array}{l}u(r, \varphi-\varphi_0)=\\=v(r, \varphi-\varphi_0)+e^{-ikr\cos(\varphi-\varphi_0)}\\u(r, \varphi+\varphi_0)=\\=v(r, \varphi+\varphi_0)+e^{-ikr\cos(\varphi+\varphi_0)}\end{array}\right\}&\text{with}\ 0<\varphi<\pi-\varphi_0,\\\left.\begin{array}{l}u(r, \varphi-\varphi_0)=\\=v(r, \varphi-\varphi_0)+e^{-ikr\cos(\varphi-\varphi_0)}\\u(r, \varphi+\varphi_0)=v(r, \varphi+\varphi_0)\end{array}\right\}&\begin{array}{c}\text{with}\\\pi-\varphi_0<\varphi<2\alpha-\pi-\varphi_0,\end{array}\\\left.\begin{array}{l}u(r, \varphi-\varphi_0)=\\=v(r, \varphi-\varphi_0)+e^{-ikr\cos(\varphi-\varphi_0)}\\u(r, \varphi+\varphi_0)=\\=v(r, \varphi+\varphi_0)+e^{-ikr\cos(2\alpha-\varphi-\varphi_0)}\end{array}\right\}&\begin{array}{c}\text{with}\\2\alpha-\pi-\varphi_0<\varphi\leqslant\alpha.\end{array}\end{array}\right| \tag{1.32}$$

The direction $\phi = \pi - \phi_0$ corresponds to the ray reflected in a specular fashion from the first face (we will consider as the first face that face from which the angles are calculated), and the direction $\varphi = 2\alpha - \pi - \varphi_0$ corresponds to the ray reflected specularly from the second face (Figure 5 and 6). The functions $e^{-ikr\cos\varphi}$ describe plane waves of unit amplitude: $e^{-ikr\cos(\varphi-\varphi_0)}$ describes the incident wave, $e^{-ikr\cos(\varphi+\varphi_0)}$ describes the wave reflected from the first face, and $e^{-ikr\cos(2\alpha-\varphi-\varphi_0)}$ — the wave reflected from the second face.

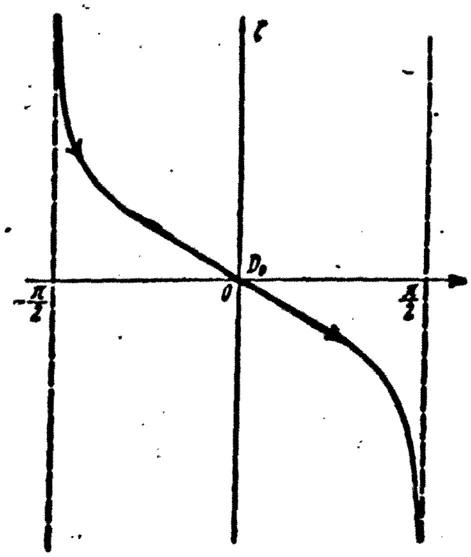


Figure 4. The integration contour in Equation (1.30).

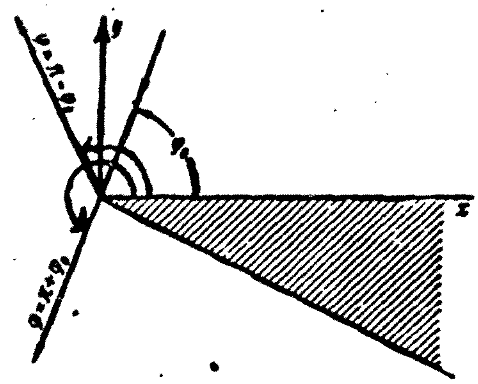


Figure 5. Diffraction of a plane wave by a wedge. The plane wave irradiates only one face of the wedge. ϕ_0 is the angle of incidence. The line $\phi = \pi - \phi_0$ is the boundary of the reflected plane wave, and the line $\phi = \pi + \phi_0$ is the boundary of the shadow.

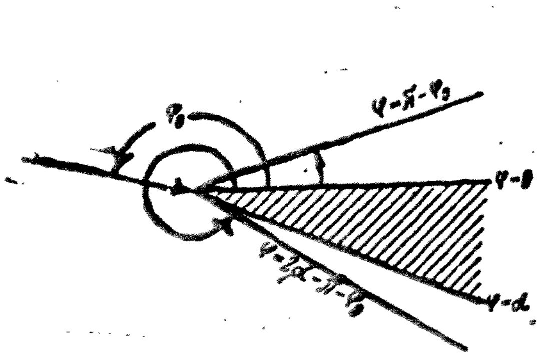


Figure 6. Diffraction by a wedge. The plane wave irradiates both faces. The line $\phi = 2\alpha - \pi - \phi_0$ is the boundary of the plane wave reflected from the second face ($\phi = \alpha$).

§ 2. Asymptotic Expressions

The integral

$$v(r,\psi)=\frac{i}{2\alpha}\sin\frac{\pi^2}{\alpha}\int\limits_{D_0}\frac{e^{ikr\cos\zeta}d\zeta}{\cos\frac{\pi^2}{\alpha}-\cos\frac{\pi}{\alpha}(\psi+\zeta)},\tag{2.01}$$

in Equations (1.31) and (1.32) generally is not expressed in terms of well-known functions. However, when kr >> 1 it may be calculated approximately by the method of steepest descents [21]. In integral (2.01), changing for this purpose to a new integration variable

$$s=\sqrt{2}e^{i\frac{\pi}{4}}\sin\frac{\zeta}{2},\ s^2=i(1-\cos\zeta),$$

we obtain

$$v(r,\psi)=\frac{\sin\frac{\pi}{n}}{\sqrt{2}\pi n}e^{i\left(kr+\frac{\pi}{4}\right)}\int\limits_{-\infty}^{\infty}\frac{e^{-krs^2}ds}{\left(\cos\frac{\pi}{n}-\cos\frac{\psi+\zeta}{n}\right)\cos\frac{\zeta}{2}},\tag{2.02}$$

where

$$n=\frac{\alpha}{\pi}.\tag{2.03}$$

It is not difficult to see that the point s = 0 is a saddle point: as one goes further from it along the imaginary axis (Re s = 0) in the plane of the complex variable s, the function e^{-krs^2} most rapidly increases, and as one goes along the real axis (Im s = 0) it decreases most rapidly. Therefore, when kr >> 1 the main contribution to the integral (2.02) is given by the integrand in the section of the contour in the vicinity of the saddle point (s = 0).

The method of steepest descents is carried out by expanding the integrand (except for the factor e^{-krs^2}) into a Taylor series in powers of s. This series is then integrated term by term. If the integrand expansion converges only on part of the integration contour, the resultant series obtained after the integration will be semiconvergent (asymptotic). Limiting ourselves to the first term in it, we obtain:

$$v(r, \psi) \approx \frac{\sin\frac{\pi}{n}}{\sqrt{2}n\pi} \frac{e^{i\left(kr+\frac{\pi}{4}\right)}}{\cos\frac{\pi}{n}-\cos\frac{\psi}{n}} \int_{-\infty}^{\infty} e^{-krs^2} ds =$$

$$= \frac{\frac{1}{n}\sin\frac{\pi}{n}}{\cos\frac{\pi}{n}-\cos\frac{\psi}{n}} \frac{e^{i\left(kr+\frac{\pi}{4}\right)}}{\sqrt{2\pi kr}}. \tag{2.04}$$

The remaining terms of the asymptotic series have a value on the order of $\frac{1}{(kr)^{3/2}}$ and higher.

Expression (2.04) is valid with the condition $\left(\cos\frac{\pi}{n}-\cos\frac{\psi}{n}\right)\sqrt{kr} \gg 1$ and describes that part of the diffraction field which has the character of cylindrical waves diverging from the wedge edge. With the incidence of the plane wave (1.23) on a wedge, the electric vector of which is parallel to the wedge edge, the cylindrical wave is determined in accordance with (1.25) and (2.04) by the equation

$$E_z = -H_\varphi = E_{0z} \cdot [v(r, \varphi-\varphi_0) - v(r, \varphi+\varphi_0)] =$$

$$= E_{0z} f \frac{e^{i\left(kr+\frac{\pi}{4}\right)}}{\sqrt{2\pi kr}}, \tag{2.05}$$

where

$$f = \frac{\sin\frac{\pi}{n}}{n}\left(\frac{1}{\cos\frac{\pi}{n}-\cos\frac{\varphi-\varphi_0}{n}} - \frac{1}{\cos\frac{\pi}{n}-\cos\frac{\varphi+\varphi_0}{n}}\right). \tag{2.06}$$

When the wedge is excited by the plane wave (1.24), in which the magnetic vector is parallel to the wedge edge, the cylindrical wave has the form

$$H_z = E_\varphi = H_{0z} \cdot [v(r, \varphi-\varphi_0) + v(r, \varphi+\varphi_0)] =$$

$$= H_{0z} g \frac{e^{i\left(kr+\frac{\pi}{4}\right)}}{\sqrt{2\pi kr}}, \tag{2.07}$$

where

$$g=\frac{\sin\frac{\pi}{n}}{n}\left(\frac{1}{\cos\frac{\pi}{n}-\cos\frac{\varphi-\varphi_0}{n}}+\frac{1}{\cos\frac{\pi}{n}-\cos\frac{\varphi+\varphi_0}{n}}\right). \tag{2.08}$$

In the vicinity of the shadow boundary ($\phi \approx \pi + \phi_0$) and near the directions of the mirror-reflected rays ($\varphi\approx\pi-\varphi_0,\ \varphi\approx 2\alpha-\pi-\varphi_0$) Expressions (2.04) - (2.08) are not valid, since the poles

$$s_1=\sqrt{2}\,e^{i\frac{\pi}{4}}\sin\frac{\pi-\psi}{2},\quad s_2=\sqrt{2}\,e^{i\frac{\pi}{4}}\sin\left(n\pi-\frac{\pi+\psi}{2}\right)$$

of the integrand in (2.02) are close to s = 0 and, consequently, its expansion in a Taylor series loses meaning. Physically, this result means that in the indicated region the diffraction wave does not reduce to plane and cylindrical waves, but has a more complicated character. An asymptotic representation of the function v (r, ψ) in this region was obtained in 1938 by Pauli [22]; here we will present the derivation of the first term of the asymptotic series obtained in [22].

Let us multiply and divide the integrand expression in Equation (2.02) by the quantity

$$\cos\psi+\cos\zeta=i\,(s^2-is_0^2)\quad\left(s_0^2=2\cos^2\frac{\psi}{2}\right) \tag{2.09}$$

and let us expand into a Taylor series in powers of s the function

$$\frac{\cos\psi+\cos\zeta}{\left(\cos\frac{\pi}{n}-\cos\frac{\psi+\zeta}{n}\right)\cos\frac{\zeta}{2}},$$

which no longer has a pole at the saddle point (s = 0) when $\psi = \phi \pm \pm \phi_0 = \pi$. Limiting ourselves in this series to the first term, we obtain

$$v(r,\psi)=\frac{\sqrt{2}}{\pi}\cdot\frac{\sin\frac{\pi}{n}}{n}\,\frac{1+\cos\psi}{\cos\frac{\pi}{n}-\cos\frac{\psi}{n}}\,e^{i\left(kr-\frac{\pi}{4}\right)}\int_{0}^{\infty}\frac{e^{-krs^2}}{s^2-is_0^2}\,ds. \tag{2.10}$$

The integral here may be represented in the form

$$\int_0^\infty \frac{e^{-krs^2}}{s^2 - is_0^2}\,ds = e^{-ikrs_0^2}\int_0^\infty ds \int_{kr}^\infty e^{-(s^2 - is_0^2)\,t}\,dt.$$

Changing the order of integration here, we find

$$\int_0^\infty \frac{e^{-krs^2}}{s^2 - is_0^2}\,ds = e^{-ikrs_0^2}\int_{kr}^\infty e^{is_0^2 t}\,\frac{dt}{\sqrt{t}}\int_0^\infty e^{-x^2}dx =$$

$$= \frac{\sqrt{\pi}}{|s_0|}\,e^{-ikrs_0^2}\int_{\sqrt{kr}\,|s_0|}^\infty e^{iq^2}dq \qquad (2.11)$$

and finally

$$v(r,\psi) = \frac{2}{n}\,\frac{\sin\frac{\pi}{n}\left|\cos\frac{\psi}{2}\right|}{\cos\frac{\pi}{n} - \cos\frac{\psi}{n}}\,e^{-ikr\cos\psi}\cdot\frac{e^{-i\frac{\pi}{4}}}{\sqrt{\pi}}\times$$

$$\times\int_{\sqrt{2kr}\left|\cos\frac{\psi}{2}\right|}^\infty e^{iq^2}dq. \qquad (2.12)$$

The next term of the asymptotic expansion for the function v (r, ψ) has a value, whose order of magnitude depends on the observation direction: in the vicinity of the border of the plane waves $(\varphi \approx \pi \pm \varphi_0)$ its order of magnitude is $\frac{1}{\sqrt{kr}}$, but far from it the order of magnitude is 1/kr in comparison with the term written in (2.12).

It is convenient to represent Expression (2.12) in the following form:

$$v(r,\psi) = -\frac{2}{n}\,\frac{\sin\frac{\pi}{n}\cos\frac{\psi}{2}}{\cos\frac{\pi}{n} - \cos\frac{\psi}{n}}\,e^{-ikr\cos\psi}\cdot\frac{e^{-i\frac{\pi}{4}}}{\sqrt{\pi}}\times$$

$$\times\int_{\infty\cos\frac{\psi}{2}}^{\sqrt{2kr}\cos\frac{\psi}{2}} e^{iq^2}dq. \qquad (2.13)$$

Here the absolute value of the lower limit of the Fresnel integral always equals infinity, and its sign is determined by the sign of $\cos \psi/2$. Therefore, when passing through the boundary of the plane waves ($\psi = \phi \pm \phi_0 = \pi$) the lower limit changes sign and the Fresnel integral undergoes a finite discontinuity, ensuring at this boundary the continuity of the function $u(r, \psi)$ and consequently of the diffraction field. Actually, by means of the well-known equation

$$\int_0^\infty e^{iq^2} dq = \frac{\sqrt{\pi}}{2} e^{i\frac{\pi}{4}} \tag{2.14}$$

it is not difficult to show that

$$v(r, \pi+0) = \frac{e^{ikr}}{2}, \quad v(r, \pi-0) = -\frac{e^{ikr}}{2} \tag{2.15}$$

and consequently

$$u(r, \pi \pm 0) = \frac{1}{2} e^{ikr}. \tag{2.16}$$

In view of the asymptotic relationships

$$\int_\infty^p e^{iq^2} dq = \frac{e^{ip^2}}{2ip}, \quad \int_{-\infty}^{-p} e^{iq^2} dq = -\frac{e^{ip^2}}{2ip} \text{ (with } p \gg 1) \tag{2.17}$$

Pauli's Equation (2.13) is transformed with $\sqrt{2kr}\left|\cos\frac{\psi}{2}\right| \gg 1$ to the Expression (2.04). As was already indicated above, it determines the cylindrical waves diverging from the wedge edge.

By means of Equation (2.13), one may also calculate the field in the vicinity of the direction $\phi = 2\alpha - \pi - \phi_0$ — that is, near the boundary of the plane wave reflected from the face $\phi = \alpha$; for this purpose, it is sufficient to replace ϕ by $\alpha - \phi$ and ϕ_0 by $\alpha - \phi_0$.

It is also interesting to note that in the case of a half-plane ($n = 2$) Equation (2.13) gives the expression

$$v(r, \psi) = e^{-ikr\cos\psi} \frac{e^{-i\frac{\pi}{4}}}{\sqrt{\pi}} \times \int_{\infty\cos\frac{\psi}{2}}^{\sqrt{2kr}\cos\frac{\psi}{2}} e^{iq^2} dq, \tag{2.18}$$

which completely agrees with the rigorous solution. Actually, with $\alpha = 2\pi$, when the wedge is transformed to a half-plane, integral (1.30) equals

$$v(r, \psi) = -\frac{i}{4\pi} \int_{D_0} \frac{e^{ikr\cos\zeta}}{\cos\frac{\psi+\zeta}{2}} d\zeta \tag{2.19}$$

and it may be reduced to a Fresnel integral. For this purpose, let us divide the contour D_0 into two parts by the point $\zeta = 0$. Summing the integrals over these parts of the contour, we find that

$$v(r, \psi) = -\frac{i}{4\pi} \int_0^{\frac{\pi}{2}-i\infty} e^{ikr\cos\zeta} \left(\frac{1}{\cos\frac{\psi+\zeta}{2}} + \frac{1}{\cos\frac{\psi-\zeta}{2}} \right) d\zeta =$$

$$= -\frac{i}{\pi} \cos\frac{\psi}{2} \int_0^{\frac{\pi}{2}-i\infty} \frac{e^{ikr\cos\zeta}\cos\frac{\zeta}{2}}{\cos\psi + \cos\zeta} d\zeta.$$

Now changing to a new integration variable $s = \sqrt{2}\, e^{i\frac{\pi}{4}} \sin\frac{\zeta}{2}$ and taking into account Equation (2.09), we obtain

$$v(r, \psi) = -\frac{\sqrt{2}}{\pi} \cos\frac{\psi}{2}\, e^{i\left(kr - \frac{\pi}{4}\right)} \int_0^{\infty} \frac{e^{-krs^2}}{s^2 - is_0^2} ds. \tag{2.20}$$

The integral here was already calculated by us. Turning to Equation (2.11), we arrive at Expression (2.18) which — together with relationships (1.25), (1.26) and (1.31) — give us the rigorous solution to the problem of the diffraction of plane waves by an ideally conducting half-plane.

§ 3. The Physical Optics Approach

In the physical optics approach, the fringing field is sought as the electro-magnetic field created by the uniform part of the surface current

$$\mathbf{j}^{0}=\frac{c}{2\pi}[\mathbf{n}\mathbf{H}_{0}]. \tag{3.01}$$

Let us recall that here n designates the external normal to the body's surface, and $\mathbf{H}_0$ designates the magnetic vector of the incident wave. First let us investigate the case $0\leqslant\varphi_0<\alpha-\pi$, when the incident plane wave irradiates only one face of the wedge (Figure 5).

From Equation (3.01) it follows that the density of the uniform part of the current being excited on the irradiated face by plane waves (1.23) and (1.24) has the following components, respectively

$$j_z^0=\frac{c}{2\pi}E_{0z}\cdot\sin\varphi_0 e^{-ikx\cos\varphi_0},\quad j_x^0=j_y^0=0 \tag{3.02}$$

and

$$j_x^0=\frac{c}{2\pi}H_{0z}e^{-ikx\cos\varphi_0},\quad j_y^0=j_z^0=0. \tag{3.03}$$

For the purpose of calculating the field radiated by this current, we will use the following integral representation of the Hankel function (see [16], p. 866)

$$H_0^{(1)}(\rho)=\frac{1}{\pi}\int\limits_{-\delta+i\infty}^{\delta-i\infty}e^{i\rho\cos\beta}d\beta=\frac{1}{i\pi}\int\limits_{-\infty}^{\infty}e^{i\rho\,\mathrm{ch}\,t}dt$$
$$(0\leqslant\delta\leqslant\pi). \tag{3.04}$$

Assuming here ρ = kd and changing to a new integration variable ζ = d sh t, we obtain

$$H_0^{(1)}(kd)=\frac{1}{i\pi}\int_{-\infty}^{\infty}\frac{e^{ik\sqrt{d^2+\zeta^2}}}{\sqrt{d^2+\zeta^2}}d\zeta. \qquad (3.05)$$

It is easy to show by means of Equations (3.02), (3.03) and (3.05) that the vector potential

$$\mathbf{A}(x,y,0)=\frac{1}{c}\int_0^{\infty}\mathbf{j}^0(\xi)\,d\xi\int_{-\infty}^{\infty}\frac{e^{ik\sqrt{y^2+(x-\xi)^2+\zeta^2}}}{\sqrt{y^2+(x-\xi)^2+\zeta^2}}d\zeta \qquad (3.06)$$

has the components

$$A_z=\frac{i}{2}E_{0z}\sin\varphi_0\cdot I_1,\quad A_x=A_y=0, \qquad (3.07)$$

if the wedge is excited by plane wave (1.23), and

$$A_x=\frac{i}{2}H_{0z}\cdot I_1,\ A_y=A_z=0, \qquad (3.08)$$

if the wedge is excited by plane wave (1.24). Here, I_1 designates the integral

$$I_1=\int_0^{\infty}e^{-ik\xi\cos\varphi_0}H_0^{(1)}(k\sqrt{y^2+(x-\xi)^2})d\xi. \qquad (3.09)$$

Let us transform it by using the relationship

$$H_0^{(1)}(k\sqrt{d^2+z^2})=\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{e^{i(vd-wz)}}{v}dw$$
$$(v=\sqrt{k^2-w^2},\ \mathrm{Im}\,v>0,\ d>0), \qquad (3.10)$$

It is not difficult to establish the correctness of this relationship by verifying that it changes into Expression (3.04) with the substitution w = k sin t, v = k cos t and $k\sqrt{d^2+z^2}=\rho$. As a result

$$I_1=\frac{1}{i\pi}\int_{-\infty}^{\infty}\frac{e^{i(v|y|-wx)}}{v(k\cos\varphi_0-w)}dw, \qquad (3.11)$$

where the integration contour passes above the pole $w = k \cos \phi_0$.

Let us note that integral (3.11) is a function of $|y|$, and let us change to polar coordinates according to the equations

$$\left.\begin{aligned} x &= r\cos\varphi, \\ |y| &= r\sin\varphi \text{ with } \varphi<\pi, \\ |y| &= -r\sin\varphi \text{ with } \varphi>\pi. \end{aligned}\right\} \tag{3.12}$$

Furthermore, by carrying out the substitution

$$w = -k\cos\xi \quad (v = k\sin\xi), \tag{3.13}$$

we obtain

$$\left.\begin{aligned} I_1 &= \frac{1}{i\pi k}\int_F \frac{e^{ikr\cos(\xi-\varphi)}}{\cos\varphi_0+\cos\xi}\,d\xi \text{ with } \varphi<\pi, \\ I_1 &= \frac{1}{i\pi k}\int_F \frac{e^{ikr\cos(\xi+\varphi)}}{\cos\varphi_0+\cos\xi}\,d\xi \text{ with } \varphi>\pi. \end{aligned}\right\} \tag{3.14}$$

The integration contour F is shown in Figures 7a and 7b. In Figure 7a the cross-hatched areas indicate the sections in the plane of the complex variable ξ in which $\operatorname{Im}\cos(\xi-\varphi)>0$; in Figure 7b, the cross-hatched areas indicate the sections where $\operatorname{Im}\cos(\xi+\varphi)>0$. Now let us deform the contour F into the contour G_1 (G_2) for the values $\varphi<\pi\,(\varphi>\pi)$, and let us change to a new integration variable

$$\left.\begin{aligned} \zeta &= \xi-\varphi && \text{with } \varphi<\pi, \\ \zeta &= \xi-(2\pi-\varphi) && \text{with } \varphi>\pi. \end{aligned}\right\} \tag{3.15}$$

As a result, we obtain the following expressions:

$$I_1 = \frac{1}{i\pi k}\int_{D_0} \frac{e^{ikr\cos\zeta}\,d\zeta}{\cos\varphi_0+\cos(\zeta+\varphi)} + \begin{cases} 0 \text{ with } \varphi>\pi-\varphi_0, \\ \dfrac{2}{k\sin\varphi_0}\, e^{-ikr\cos(\varphi+\varphi_0)} \\ \qquad \text{with } \varphi<\pi-\varphi_0, \end{cases} \tag{3.16}$$

if $\phi < \pi$ and

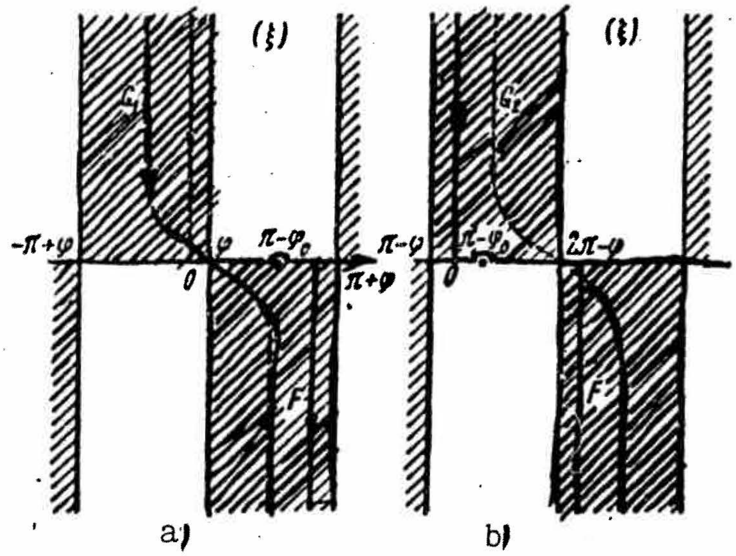


Figure 7. The integration contour in Equations (3.14).

$$I_1 = \frac{1}{i\pi k}\int_{D_0} \frac{e^{ikr\cos\zeta}d\zeta}{\cos\varphi_0 + \cos(\zeta-\varphi)} + \begin{cases} 0 \text{ with } \varphi < \pi+\varphi_0, \\ \frac{2}{k\sin\varphi_0} e^{-ikr\cos(\varphi-\varphi_0)} \\ \qquad \text{with } \varphi > \pi+\varphi_0, \end{cases} \tag{3.17}$$

if $\phi > \pi$. The integration contour D_0 is shown in Figure 4.

By means of Equation (3.06) and the equality

$$E_z = ikA_z \tag{3.18}$$

let us find the field which is radiated by the uniform part of the current excited on the face $\phi = 0$ by the plane wave (1.23)

$$\frac{E_z}{E_{0z}} = \begin{cases} v_1^+(\varphi,\varphi_0) - e^{-ikr\cos(\varphi+\varphi_0)} \text{ with } 0<\varphi<\pi-\varphi_0, \\ v_1^+(\varphi,\varphi_0) \qquad \text{with } \pi-\varphi_0<\varphi<\pi, \\ v_1^-(\varphi,\varphi_0) \qquad \text{with } \pi<\varphi<\pi+\varphi_0, \\ v_1^-(\varphi,\varphi_0) - e^{-ikr\cos(\varphi-\varphi_0)} \text{ with } \pi+\varphi_0<\varphi\leq\alpha, \end{cases} \tag{3.19}$$

where

$$v_1^{\pm}(\varphi, \varphi_0) = \frac{i}{2\pi} \sin\varphi_0 \int\limits_{D_0} \frac{e^{ikr\cos\zeta} d\zeta}{\cos\varphi_0 + \cos(\zeta \pm \varphi)}. \tag{3.20}$$

It is not difficult to see that with $\alpha - \pi < \varphi_0 < \pi$, when both faces of the wedge are illuminated, the field excited by the second face ($\phi = \alpha$) will be described by the same equations if one replaces ϕ by $\alpha - \phi$, and ϕ_0 by $\alpha - \phi_0$ in them.

Adding the field being radiated by the uniform part of the current with the incident plane wave (1.23), we obtain the diffraction field in the physical optics approach. It equals

$$\frac{E_z}{E_{0z}} = \begin{cases} v_1^{+}(\varphi, \varphi_0) + e^{-ikr\cos(\varphi - \varphi_0)} - e^{-ikr\cos(\varphi + \varphi_0)} \\ \qquad\qquad \text{with } 0 \leqslant \varphi < \pi - \varphi_0, \\ v_1^{+}(\varphi, \varphi_0) + e^{-ikr\cos(\varphi - \varphi_0)} \\ \qquad\qquad \text{with } \pi - \varphi_0 < \varphi < \pi, \\ v_1^{-}(\varphi, \varphi_0) + e^{-ikr\cos(\varphi - \varphi_0)} \\ \qquad\qquad \text{with } \pi < \varphi < \pi + \varphi_0, \\ v_1^{-}(\varphi, \varphi_0) \qquad \text{with } \pi + \varphi_0 < \varphi < \alpha, \end{cases} \tag{3.21}$$

if one face of the wedge $(0 \leqslant \varphi_0 < \alpha - \pi)$ is illuminated, and

$$\frac{E_z}{E_{0z}} = \begin{cases} v_1^{+}(\varphi, \varphi_0) + v_1^{-}(\alpha - \varphi, \alpha - \varphi_0) + \\ + e^{-ikr\cos(\varphi - \varphi_0)} - e^{-ikr\cos(\varphi + \varphi_0)} \\ \qquad\qquad \text{with } 0 \leqslant \varphi < \pi - \varphi_0, \\ v_1^{+}(\varphi, \varphi_0) + v_1^{-}(\alpha - \varphi, \alpha - \varphi_0) + \\ + e^{-ikr\cos(\varphi - \varphi_0)} \quad \text{with } \pi - \varphi_0 < \varphi < \alpha - \pi, \\ v_1^{+}(\varphi, \varphi_0) + v_1^{+}(\alpha - \varphi, \alpha - \varphi_0) + \\ + e^{-ikr\cos(\varphi - \varphi_0)} \quad \text{with } \alpha - \pi < \varphi < \pi, \\ v_1^{-}(\varphi, \varphi_0) + v_1^{+}(\alpha - \varphi, \alpha - \varphi_0) + \\ + e^{-ikr\cos(\varphi - \varphi_0)} \quad \text{with } \pi < \varphi < 2\alpha - \pi - \varphi_0, \\ v_1^{-}(\varphi, \varphi_0) + v_1^{+}(\alpha - \varphi, \alpha - \varphi_0) + \\ + e^{-ikr\cos(\varphi - \varphi_0)} - e^{-ikr\cos(2\alpha - \varphi - \varphi_0)} \\ \qquad\qquad \text{with } 2\alpha - \pi - \varphi_0 < \varphi \leqslant \alpha, \end{cases} \tag{3.22}$$

if both faces of the wedge $(\alpha-\pi<\varphi_0<\pi)$ are illuminated.

Now let us calculate the field arising during diffraction by a wedge of a plane wave (1.24). The field scattered by the first face (ϕ = 0) is determined by the relationship

$$\left.\begin{aligned} H_z &= -\frac{\partial A_z}{\partial y}, \\ H_x &= H_y = 0. \end{aligned}\right\} \tag{3.23}$$

One may write the component H_z in the form

$$H_z = -\frac{i}{2} H_{0z} \frac{\partial}{\partial y} I_1 \tag{3.24}$$

or

$$\left.\begin{aligned} H_z &= -\frac{i}{2} H_{0z} \cdot I_2 \text{ with } \varphi<\pi, \\ H_z &= \frac{i}{2} H_{0z} \cdot I_2 \text{ with } \varphi>\pi. \end{aligned}\right\} \tag{3.25}$$

The quantity I_2 introduced here is the integral

$$I_2 = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{e^{i(v|y|-wx)}}{k\cos\varphi_0 - w}\, dw \tag{3.26}$$

along the infinite contour which passes above the pole $w = k \cos \phi_0$. This integral, precisely the same as integral I_1, is transformed to an integral along the contour D_0. As a result, we obtain

$$\frac{H_z}{H_{0z}} = \begin{cases} v_2^+(\varphi, \varphi_0) + e^{-ikr\cos(\varphi+\varphi_0)} & \text{with } 0 \leq \varphi < \pi - \varphi_0, \\ v_2^+(\varphi, \varphi_0) & \text{with } \pi-\varphi_0<\varphi<\pi, \\ v_2^-(\varphi, \varphi_0) & \text{with } \pi<\varphi<\pi+\varphi_0, \\ v_2^-(\varphi, \varphi_0) - e^{-ikr\cos(\varphi-\varphi_0)} & \text{with } \pi+\varphi_0<\varphi\leq\alpha, \end{cases} \tag{3.27}$$

where

$$v_2^{\pm}(\varphi, \varphi_0) = \mp \frac{i}{2\pi} \int_{D_0} \frac{\sin(\zeta \pm \varphi)\, e^{ikr\cos\zeta}}{\cos\varphi_0 + \cos(\zeta \pm \varphi)}\, d\zeta. \tag{3.28}$$

In the case when both faces of the wedge are illuminated, the field scattered by the second face also is determined by Equations (3.27) and (3.28) in which one need only replace ϕ_0 by $\alpha - \phi_0$, and ϕ by $\alpha - \phi$.

Then adding the field radiated by the nonuniform part of the current with the incident wave (1.24), we find the diffraction field in the physical optics approach. This equals

$$\frac{H_z}{H_{0z}} = \begin{cases} v_2^+(\varphi, \varphi_0) + e^{-ikr\cos(\varphi - \varphi_0)} + e^{-ikr\cos(\varphi + \varphi_0)} & \text{with } 0 \leqslant \varphi < \pi - \varphi_0, \\ v_2^+(\varphi, \varphi_0) + e^{-ikr\cos(\varphi - \varphi_0)} & \text{with } \pi - \varphi_0 < \varphi < \pi, \\ v_2^-(\varphi, \varphi_0) + e^{-ikr\cos(\varphi - \varphi_0)} & \text{with } \pi < \varphi < \pi + \varphi_0, \\ v_2^-(\varphi, \varphi_0) & \text{with } \pi + \varphi_0 < \varphi \leqslant \alpha, \end{cases} \tag{3.29}$$

if one face of the wedge is illuminated, and

$$\frac{H_z}{H_{0z}} = \begin{cases} v_2^+(\varphi, \varphi_0) + v_2^-(\alpha - \varphi, \alpha - \varphi_0) + \\ + e^{-ikr\cos(\varphi - \varphi_0)} + e^{-ikr\cos(\varphi + \varphi_0)} & \text{with } 0 \leqslant \varphi \leqslant \pi - \varphi_0, \\ v_2^+(\varphi, \varphi_0) + v_2^-(\alpha - \varphi, \alpha - \varphi_0) + \\ + e^{-ikr\cos(\varphi - \varphi_0)} & \text{with } \pi - \varphi_0 < \varphi < \alpha - \pi, \\ v_2^+(\varphi, \varphi_0) + v_2^+(\alpha - \varphi, \alpha - \varphi_0) + \\ + e^{-ikr\cos(\varphi - \varphi_0)} & \text{with } \alpha - \pi < \varphi < \pi, \\ v_2^-(\varphi, \varphi_0) + v_2^+(\alpha - \varphi, \alpha - \varphi_0) + \\ + e^{-ikr\cos(\varphi - \varphi_0)} & \text{with } \pi < \varphi < 2\alpha - \pi - \varphi_0, \\ v_2^-(\varphi, \varphi_0) + v_2^+(\alpha - \varphi, \alpha - \varphi_0) + \\ + e^{-ikr\cos(\varphi - \varphi_0)} + e^{-ikr\cos(2\alpha - \varphi - \varphi_0)} & \text{with } 2\alpha - \pi - \varphi_0 < \varphi \leqslant \alpha, \end{cases} \tag{3.30}$$

if both of its faces are illuminated.

The integrals $V_1^{\pm}$, $V_2^{\pm}$ generally are not expressed in terms of well-known functions. However, by using the method of steepest descents, it is not difficult to obtain their asymptotic expansion when $kr >> 1$. Far from the directions $\phi = \pi \pm \phi_0$ and $\phi = 2\alpha - \pi - \phi_0$, the first term of the asymptotic expansion gives us the cylindrical wave diverging from the wedge edge. In the case of wedge excitation by a plane wave (1.23), these cylindrical waves are determined by the equation

$$\left.\begin{aligned} E_z = -H_\varphi = E_{0z}\cdot f^0\cdot\frac{e^{i\left(kr+\frac{\pi}{4}\right)}}{\sqrt{2\pi kr}}, \\ E_\varphi = H_z = 0, \end{aligned}\right\} \qquad (3.31)$$

and with the excitation of the wedge by a plane wave (1.24) they are determined by the equation

$$\left.\begin{aligned} H_z = E_\varphi = H_{0z}\cdot g^0\cdot\frac{e^{i\left(kr+\frac{\pi}{4}\right)}}{\sqrt{2\pi kr}}, \\ H_\varphi = E_z = 0. \end{aligned}\right\} \qquad (3.32)$$

The functions f^0 and g^0 have the form

$$\left.\begin{aligned} f^0 = \frac{\sin\varphi_0}{\cos\varphi + \cos\varphi_0}, \\ g^0 = -\frac{\sin\varphi}{\cos\varphi + \cos\varphi_0}, \end{aligned}\right\} \qquad (3.33)$$

if one face of the wedge $(0 < \varphi_0 < \alpha - \pi)$, and

$$\left.\begin{aligned} f^0 = \frac{\sin\varphi_0}{\cos\varphi + \cos\varphi_0} + \frac{\sin(\alpha-\varphi_0)}{\cos(\alpha-\varphi)+\cos(\alpha-\varphi_0)}, \\ g^0 = -\frac{\sin\varphi}{\cos\varphi + \cos\varphi_0} - \frac{\sin(\alpha-\varphi)}{\cos(\alpha-\varphi)+\cos(\alpha-\varphi_0)}, \end{aligned}\right\} \qquad (3.34)$$

if both of its faces $(\alpha - \pi < \varphi_0 < \pi)$ are illuminated. The index "0" for the functions f^0 and g^0 means that the cylindrical waves (3.31) and (3.32) are radiated by the uniform part of the surface current (j^0).

§ 4. The Field Radiated by the Nonuniform Part of the Current

In § 1 and 3 we represented the rigorous and approximate expressions for the diffraction field by integrals along the same contour in the complex variable plane. By subtracting the approximate expression from the rigorous expression, we find the field created by the nonuniform part of the current. It is determined by integrals of the type

$$\int_{D_0} p(\alpha, \varphi, \varphi_0, \zeta)\, e^{ikr\cos\zeta} d\zeta, \tag{4.01}$$

which, with the replacement of the variable ζ by $s = \sqrt{2}e^{i\frac{\pi}{4}} \sin\frac{\zeta}{2}$, are transformed to the form

$$e^{ikr} \int_{-\infty}^{\infty} q(\alpha, \varphi, \varphi_0, s)\, e^{-krs^2} ds \tag{4.02}$$

and may be approximately calculated by the method of steepest descents.

For this purpose, let us expand the function q(s) into a Taylor series

$$q(\alpha, \varphi, \varphi_0, s) = q_0 + q_1 \cdot s + q_2 \cdot s^2 + \ldots \tag{4.03}$$

Let us note that expansion (4.03) does not have meaning only in the particular case

$$\left.\begin{array}{ll} \varphi = \pi \pm \varphi_0 & \text{with } \varphi_0 = 0;\ \pi, \\ \varphi = 2\alpha - \pi - \varphi_0 & \text{with } \varphi_0 = \alpha - \pi, \end{array}\right\} \tag{4.04}$$

when the observation direction (ϕ) coincides with the direction of propagation of the incident wave glancing along one of the wedge face

Substituting series (4.03) into Equation (4.02) and then performing a term by term integration, we find the asymptotic expansion for the field radiated by the nonuniform part of the current. We limit ourselves to the first term of the asymptotic expansion, omitting terms of the order $(kr)^{-3/2}$ and higher. As a result, the required field from the nonuniform part of the current will equal

$$\left.\begin{aligned} &E_z = -H_\varphi = E_{0z} f^1 \frac{e^{i\left(kr + \frac{\pi}{4}\right)}}{\sqrt{2\pi kr}}, \\ &E_\varphi = H_z = 0 \end{aligned}\right\} \tag{4.05}$$

with wedge excitation by plane wave (1.23), and

$$\left.\begin{aligned} &H_z = E_\varphi = H_{0z} g^1 \frac{e^{i\left(kr + \frac{\pi}{4}\right)}}{\sqrt{2\pi kr}}, \\ &H_\varphi = E_z = 0 \end{aligned}\right\} \tag{4.06}$$

with wedge excitation by plane wave (1.24).

By calculating, with the help of Equations (4.05) and (4.06), the nonuniform part of the current, it is not difficult to see that it is concentrated mainly in the vicinity of the wedge edge. But the field created in the region kr >> 1 by this part of the current has the form of cylindrical waves, the angular functions of which are determined by the relationships(1)

$$f^1 = f - f^0, \quad g^1 = g - g^0, \tag{4.07}$$

where in accordance with § 1 and 3 we have

$$\left.\begin{aligned} f \\ g \end{aligned}\right\} = \frac{\sin\frac{\pi}{n}}{n}\left(\frac{1}{\cos\frac{\pi}{n} - \cos\frac{\varphi - \varphi_0}{n}} \mp \frac{1}{\cos\frac{\pi}{n} - \cos\frac{\varphi + \varphi_0}{n}}\right)$$
$$\left(n = \frac{\alpha}{\pi}\right) \tag{4.08}$$

(1) Footnote appears on page 42.

and

$$\left.\begin{aligned} f^0 &= \frac{\sin\varphi_0}{\cos\varphi+\cos\varphi_0}, \\ g^0 &= -\frac{\sin\varphi}{\cos\varphi+\cos\varphi_0}, \end{aligned}\right\} \qquad (4.09)$$

if one face of the wedge is illuminated (that is, when $0<\varphi_0<\alpha-\pi$), and

$$\left.\begin{aligned} f^0 &= \frac{\sin\varphi_0}{\cos\varphi+\cos\varphi_0}+\frac{\sin(\alpha-\varphi_0)}{\cos(\alpha-\varphi)+\cos(\alpha-\varphi_0)}, \\ g^0 &= -\frac{\sin\varphi}{\cos\varphi+\cos\varphi_0}-\frac{\sin(\alpha-\varphi)}{\cos(\alpha-\varphi)+\cos(\alpha-\varphi_0)}, \end{aligned}\right\} \qquad (4.10)$$

if both faces of the wedge are illuminated (that is, when $\alpha-\pi<\varphi_0<\pi$). Let us recall that the functions f and g describe the cylindrical waves radiated by the total current — that is, the sum of the uniform and nonuniform parts, and the functions f^0 and g^0 refer to the cylindrical waves radiated only by the uniform part of the current (j^0).

Let us note certain properties of the functions f^1 and g^1. The function $f^1 = f^1(\alpha, \phi, \phi_0)$ is continuous, whereas the function $g^1 = g^1(\alpha, \phi, \phi_0)$ undergoes a finite discontinuity when $\phi_0 = \alpha - \pi$. The reason for this discontinuity is that the uniform part of the current differs from zero on the face along which plane wave (1.24) is propagated (with $\phi_0 = \alpha - \pi$). In the case of radar, when the direction to the observation point coincides with the direction to the source ($\phi = \phi_0$), both functions f^1 and g^1 are continuous. There is no discontinuity of the function g^1 with $\phi = \phi_0 = \alpha - \pi$, because the current element does not radiate in the longitudinal direction.

On the boundary of the plane waves (that is, when $\varphi=\pi\pm\varphi_0$ and $\varphi=2\alpha-\pi-\varphi_0$) the functions f, f^0 and g, g^0 become infinite, whereas the functions f^1 and g^1 remain finite. In accordance with Equations (4.07) - (4.10), they take the following values

$$\left.\begin{matrix} f^1 \\ g^1 \end{matrix}\right\} = \frac{\frac{1}{n}\sin\frac{\pi}{n}}{\cos\frac{\pi}{n} - \cos\frac{\varphi-\varphi_0}{n}} + \frac{1}{2}\operatorname{ctg}\varphi_0 \pm \frac{1}{2n}\operatorname{ctg}\frac{\pi}{n}, \tag{4.11}$$

if $\phi = \pi - \phi_0$, and $\phi_0 < \alpha - \pi$,

$$\left.\begin{aligned} f^1 &= \frac{\frac{1}{n}\sin\frac{\pi}{n}}{\cos\frac{\pi}{n} - \cos\frac{\varphi-\varphi_0}{n}} + \frac{1}{2}\operatorname{ctg}\varphi_0 + \frac{1}{2n}\operatorname{ctg}\frac{\pi}{n} - \\ &\quad - \frac{\sin(\alpha-\varphi_0)}{\cos(\alpha-\varphi)+\cos(\alpha-\varphi_0)}, \\ g^1 &= \frac{\frac{1}{n}\sin\frac{\pi}{n}}{\cos\frac{\pi}{n} - \cos\frac{\varphi-\varphi_0}{n}} + \frac{1}{2}\operatorname{ctg}\varphi_0 - \frac{1}{2n}\operatorname{ctg}\frac{\pi}{n} + \\ &\quad + \frac{\sin(\alpha-\varphi)}{\cos(\alpha-\varphi)+\cos(\alpha-\varphi_0)}, \end{aligned}\right\} \tag{4.12}$$

if $\phi = \pi - \phi_0$ and $\alpha - \pi < \phi_0 < \pi$, and

$$\left.\begin{matrix} f^1 \\ g^1 \end{matrix}\right\} = \mp \frac{\frac{1}{n}\sin\frac{\pi}{n}}{\cos\frac{\pi}{n} - \cos\frac{\varphi+\varphi_0}{n}} \pm \frac{1}{2}\operatorname{ctg}\varphi_0 - \frac{1}{2n}\operatorname{ctg}\frac{\pi}{n}, \tag{4.13}$$

if $\phi = \pi + \phi_0$, and $\phi_0 < \alpha - \pi$. The value $\phi = \pi + \phi_0$ with $\alpha - \pi < \phi_0 < \pi$ corresponds to the angle inside the wedge, and therefore is not of interest. In the direction of the mirror-reflected ray $\phi = 2\alpha - \pi - \phi_0$, the functions f^1 and g^1 are determined (with $\alpha - \pi < \phi_0 < \pi$) by the following equation:

$$\begin{aligned} f^1 &= \frac{\frac{1}{n}\sin\frac{\pi}{n}}{\cos\frac{\pi}{n} - \cos\frac{\varphi-\varphi_0}{n}} - \frac{\sin\varphi_0}{\cos\varphi+\cos\varphi_0} + \\ &\quad + \frac{1}{2}\operatorname{ctg}(\alpha-\varphi_0) + \frac{1}{2n}\operatorname{ctg}\frac{\pi}{n}, \\ g^1 &= \frac{\frac{1}{n}\sin\frac{\pi}{n}}{\cos\frac{\pi}{n} - \cos\frac{\varphi-\varphi_0}{n}} + \frac{\sin\varphi}{\cos\varphi+\cos\varphi_0} + \\ &\quad + \frac{1}{2}\operatorname{ctg}(\alpha-\varphi_0) - \frac{1}{2n}\operatorname{ctg}\frac{\pi}{n}. \end{aligned} \tag{4.14}$$

The functions f^1 and g^1 have a finite value everywhere, except for the particular values ϕ and ϕ_0 enumerated in Equation (4.04). The graphs of the functions f^1 and g^1 (Figures 8 - 13) drawn in polar coordinates give a visual representation of the effect of the nonuniform part of the current which is concentrated near the wedge edge. In particular, they show that this effect may be usbstantial for the fringing field not only in the shadow region $(\pi+\varphi_0<\varphi\leqslant\alpha)$, but also in the region of light $(0\leqslant\varphi<\pi+\varphi_0)$. The continuous lines in the figures correspond to the functions f^1 $(f^1 < 0)$. The dashed and dash-dot lines correspond to the functions g^1 — the dash lines refer to the case $g^1 < 0$, and the dash-dot lines refer to the case $g^1 > 0$.

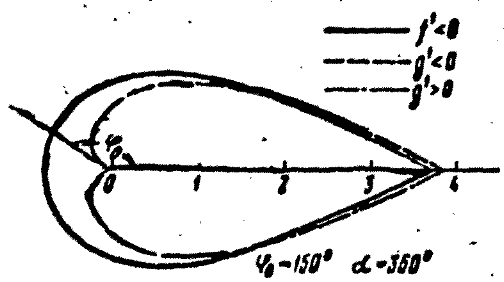


Figure 8. The diagram of the field from the nonuniform part of the current excited by a plane wave on a half-plane. The function f^1 (or g^1) corresponds to the case when the electric (or magnetic) vector of the incident wave is parallel to the wedge edge.

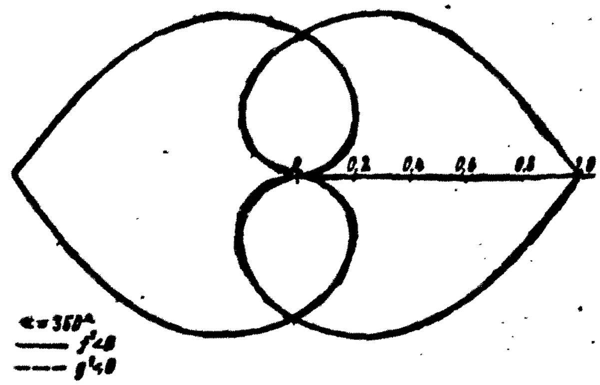


Figure 9. The same as Figure 8 for the case $\phi = \phi_0$.

Let us turn our attention to the next important aspect. As is seen from § 1 and 3, the nonuniform part of the current on the wedge is described by a contour integral which is generally not expressed in terms of well-known functions. But in order to calculate the field scattered by some convex, ideally conducting surface with discontinuities (edges), the indicated expression still must be integrated over the given surface. Obviously, such a path is able to lead only to very cumbersome equations. Therefore, henceforth, when calculating

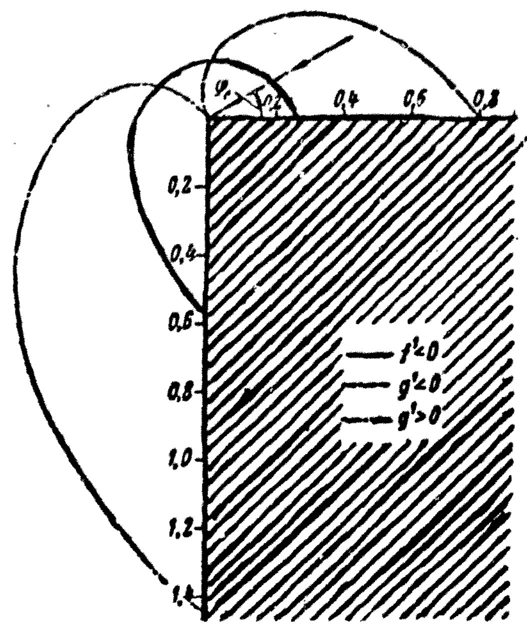


Figure 10. The functions f^1 and g^1 for a wedge (ϕ_0 = 30°, α = 270°).

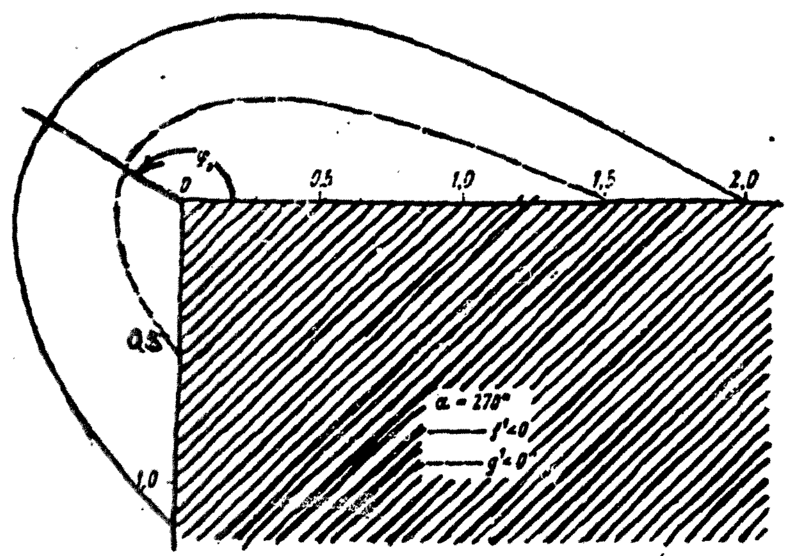
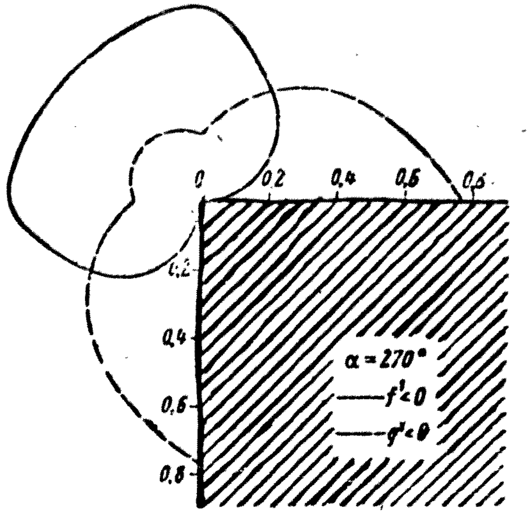


Figure 11. The same as Figure 10 when ϕ_0 = 150°.



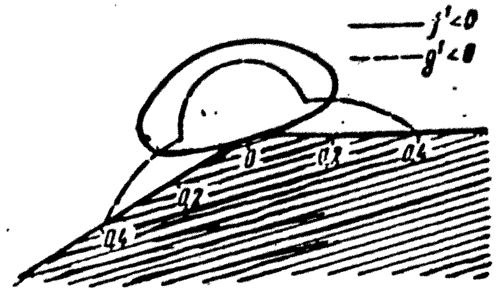


Figure 12. The same as Figure 10 for the case $\phi = \phi_0$.

Figure 13. The functions f^1 and g^1 for a wedge ($\phi = \phi_0$, $\alpha = 210°$).

the field scattered by composite bodies, we will not integrate the explicit expressions for the nonuniform part of the current, but we will endeavour to express these integrals directly in terms of the functions f^1 and g^1 which have been found.

§ 5. The Oblique Incidence of a Plane Wave on a Wedge

Above, the diffraction was studied of a plane wave incident on a wedge perpendicular to its edge. Now let us investigate the case when the plane wave

$$E = E_0 e^{ik(x\cos\alpha + y\cos\beta + z\cos\gamma)} \tag{5.01}$$

falls on the wedge at an oblique angle γ $\left(0 < \gamma < \frac{\pi}{2}\right)$ to the wedge edge (Figure 14).

From the geometry of the problem, it follows that the diffraction field must have that same dependence on the z coordinate as the field of the incident wave, that is

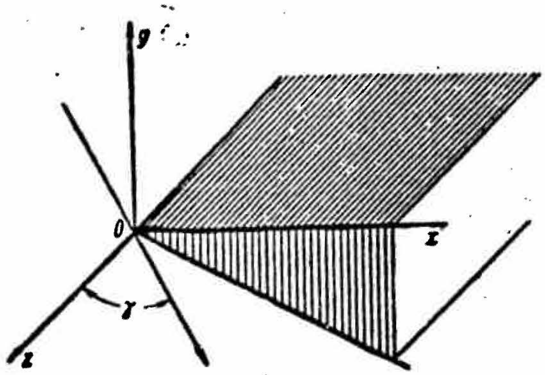


Figure 14. Diffraction by a wedge with oblique incidence of a plane wave. γ is the angle between the normal to the incident wave front and the z axis.

$$\left.\begin{aligned} \mathbf{E} &= \mathbf{E}(x, y)\, e^{ikz\cos\gamma}, \\ \mathbf{H} &= \mathbf{H}(x, y)\, e^{ikz\cos\gamma}. \end{aligned}\right\} \qquad (5.02)$$

Using Maxwell's equations

$$\operatorname{rot}\mathbf{H} = -ik\mathbf{E}, \quad \operatorname{rot}\mathbf{E} = ik\mathbf{H}, \qquad (5.03)$$

one is able to obtain the following expressions for the radial and azimuthal components of the field:

$$\begin{aligned} E_r &= -\frac{1}{ik\sin^2\gamma}\left(\frac{1}{r}\frac{\partial H_z}{\partial\varphi} + \cos\gamma\frac{\partial E_z}{\partial r}\right), \\ H_r &= \frac{1}{ik\sin^2\gamma}\left(\frac{1}{r}\frac{\partial E_z}{\partial\varphi} - \cos\gamma\frac{\partial H_z}{\partial r}\right), \\ E_\varphi &= \frac{1}{ik\sin^2\gamma}\left(\frac{\partial H_z}{\partial r} - \frac{\cos\gamma}{r}\frac{\partial E_z}{\partial\varphi}\right), \\ H_\varphi &= -\frac{1}{ik\sin^2\gamma}\left(\frac{\partial E_z}{\partial r} + \frac{\cos\gamma}{r}\frac{\partial H_z}{\partial\varphi}\right). \end{aligned} \qquad (5.04)$$

The functions E_z and H_z in turn satisfy the wave equations

$$\Delta E_z + k_1^2 E_z = 0, \quad \Delta H_z + k_1^2 H_z = 0, \qquad (5.05)$$

where

$$\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \text{ and } k_1 = k\sin\gamma. \qquad (5.06)$$

In § 4 we found the fields (4.05) and (4.06) which satisfy the equations

$$\Delta E_z + k^2 E_z = 0, \quad \Delta H_z + k^2 H_z = 0 \qquad (5.07)$$

and which are created by the nonuniform part of the current excited on the wedge by the plane wave

$$E = E_0 e^{-ik(x\cos\varphi_0 + y\sin\varphi_0)}. \tag{5.08}$$

Representing Expression (5.01) in the form

$$E = E_0 e^{ikz\cos\gamma - ik_1(x\cos\varphi_0 + y\sin\varphi_0)} \tag{5.09}$$

and comparing Equations (5.05) and (5.07), we easily find the field created by the nonuniform part of the current with the irradiation of the wedge by plane wave (5.01). For this purpose, it is sufficient to replace in Equations (4.05) and (4.06) k by k_1, and E_{0z} and H_{0z} by $E_{0z}e^{ikz\cos\gamma}$ and $H_{0z}e^{ikz\cos\gamma}$. As a result, we obtain

$$\left.\begin{aligned} E_z = -H_\varphi = E_{0z} f^1(\varphi, \varphi_0) \frac{e^{i\left(k_1 r + \frac{\pi}{4}\right)}}{\sqrt{2\pi k_1 r}} e^{ikz\cos\gamma}, \\ H_z = E_\varphi = H_{0z} g^1(\varphi, \varphi_0) \frac{e^{i\left(k_1 r + \frac{\pi}{4}\right)}}{\sqrt{2\pi k_1 r}} e^{ikz\cos\gamma}. \end{aligned}\right\} \tag{5.10}$$

The angle ϕ_0 introduced here is determined by the condition

$$e^{ik(x\cos\alpha + y\cos\beta)} = e^{-ik_1(x\cos\varphi_0 + y\sin\varphi_0)}, \tag{5.11}$$

hence

$$\operatorname{tg}\varphi_0 = \frac{\cos\beta}{\cos\alpha}. \tag{5.12}$$

The remaining components of the field created by the nonuniform part of the current with the oblique incidence of a plane wave are found from relationships (5.04), and when kr >> 1 they equal

$$\begin{aligned} E_r = -\operatorname{ctg}\gamma E_z \quad H_r = -\operatorname{ctg}\gamma H_z \\ E_\varphi = \frac{1}{\sin\gamma} H_z \quad H_\varphi = -\frac{1}{\sin\gamma} E_z. \end{aligned} \tag{5.13}$$

The equiphase surfaces for these waves have the form

$$r\sin\gamma + z\cos\gamma = \mathrm{const} \tag{5.14}$$

and are conical surfaces, the generatrices of which form the angle $\pi/2 + \gamma$ with the positive direction of the z axis. Thus, with oblique irradiation of the wedge by a plane wave, the field created by the nonuniform part of the current is a set of conical waves diverging from the wedge edge. The normals to the phase surfaces of these waves form an angle γ with the positive direction of the z axis and are shown in Figure 15. These waves may be represented in a more graphic form if one introduces the components (see Figure 15):

$$\left.\begin{aligned} E_\gamma &= E_r\cos\gamma - E_z\sin\gamma, \\ H_\gamma &= H_r\cos\gamma - H_z\sin\gamma. \end{aligned}\right\} \qquad (5.15)$$

Then the final expressions for the fringing field in the far zone will have the form

$$\left.\begin{aligned} E_\gamma &= H_\varphi = -\frac{1}{\sin\gamma}E_z, \\ H_\gamma &= -E_\varphi = -\frac{1}{\sin\gamma}H_z. \end{aligned}\right\} \qquad (5.16)$$

Now we are able to proceed to the application of the results which have been obtained for the solution of specific diffraction problems. The simplest of them is the problem of diffraction by an infinitely long strip which has a rigorous solution [23] in the form of Mathieu function series. However, in the quasi-optical region when the width of the strip is large in comparison with the wavelength, these series have a poor convergence and are not suitable for numerical calculations. Therefore, the requirement arises for approximation equations which are useful in the quasi-optical region. The derivation of such equations for a field scattered by a strip will be given in the following section.

§ 6. Diffraction by a Strip

Let us investigate diffraction by an infinitely thin, ideally conducting strip which has a width of 2a and an unlimited length. The orientation of the strip in space is shown in Figure 16.

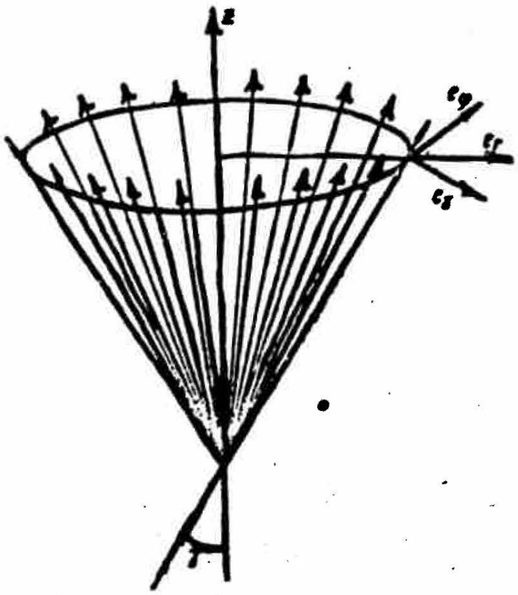


Figure 15. The cone of diffracted rays.

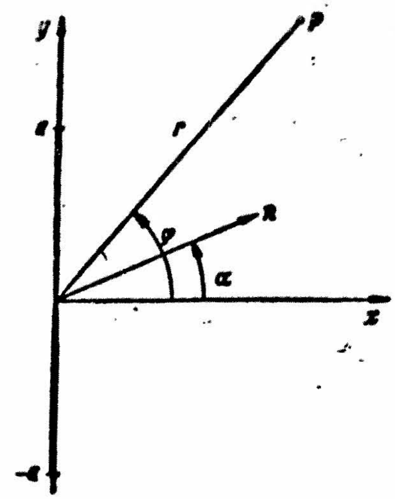


Figure 16. Diffraction of a plane wave by an infinitely long strip. The section of the axis y ($-a \leq y \leq a$) shows the transverse cross section of the strip with the plane z = 0; α is the angle of incidence.

Let a plane, electromagnetic wave strike the strip perpendicular to the edges. Let the direction of propagation of this wave form an angle $\alpha\left(|\alpha| < \frac{\pi}{2}\right)$ with the plane y = 0. The field of this wave is represented in the form

$$\mathbf{E} = \mathbf{E}_0 e^{ik(x\cos\alpha + y\sin\alpha)}, \quad \mathbf{H} = \mathbf{H}_0 e^{ik(x\cos\alpha + y\sin\alpha)}. \tag{6.01}$$

The uniform part of the current excited by the plane wave on the strip has the components

$$\left.\begin{aligned} j_x^0 &= 0, \\ j_y^0 &= \frac{c}{2\pi} H_{0z} e^{iky\sin\alpha}, \\ j_z^0 &= \frac{c}{2\pi} E_{0z}\cos\alpha\, e^{iky\sin\alpha}. \end{aligned}\right\} \tag{6.02}$$

Substituting these values into the equation for the vector potential

$$\mathbf{A}=\frac{1}{c}\int_{-a}^{a}\mathbf{j}^{0}(\eta)\,d\eta\int_{-\infty}^{\infty}\frac{e^{ik\sqrt{x^2+(y-\eta)^2+\zeta^2}}}{\sqrt{x^2+(y-\eta)^2+\zeta^2}}\,d\zeta \tag{6.03}$$

and taking into account relationships (3.05) and (1.18), we obtain the following expressions in the region $r \gg ka^2$:

$$\left.\begin{aligned}
A_x &= 0,\\
A_y &= \frac{2}{k}H_{0z}\frac{\sin[ka(\sin\alpha-\sin\varphi)]}{\sin\alpha-\sin\varphi}\,\frac{e^{i\left(kr+\frac{\pi}{4}\right)}}{\sqrt{2\pi kr}},\\
A_z &= \frac{2}{k}E_{0z}\cos\alpha\,\frac{\sin[ka(\sin\alpha-\sin\varphi)]}{\sin\alpha-\sin\varphi}\,\frac{e^{i\left(kr+\frac{\pi}{4}\right)}}{\sqrt{2\pi kr}}.
\end{aligned}\right\} \tag{6.04}$$

The components of the fringing field in the cylindrical coordinate system equal

$$E_z=-H_\varphi=ikA_z,\quad H_z=E_\varphi=ikA_\varphi, \tag{6.05}$$

where

$$A_\varphi=A_y\cos\varphi-A_x\sin\varphi. \tag{6.06}$$

Substituting Expressions (6.04) here, let us determine the field radiated by the uniform part of the current

$$\left.\begin{aligned}
E_z &= -H_\varphi=2E_{0z}\cos\alpha\,\frac{\sin[ka(\sin\alpha-\sin\varphi)]}{\sin\alpha-\sin\varphi}\,\frac{e^{i\left(kr+\frac{3\pi}{4}\right)}}{\sqrt{2\pi kr}},\\
E_\varphi &= H_z=2H_{0z}\cos\varphi\,\frac{\sin[ka(\sin\alpha-\sin\varphi)]}{\sin\alpha-\sin\varphi}\,\frac{e^{i\left(kr+\frac{3\pi}{4}\right)}}{\sqrt{2\pi kr}}.
\end{aligned}\right\} \tag{6.07}$$

This field may be represented in the form of cylindrical waves diverging from the strip edges

$$
\left.
\begin{aligned}
E_z = -H_\varphi = E_{0z}\cdot\Big[f^0(1)\,e^{ika(\sin\alpha-\sin\varphi)} + {} \\
+ f^0(2)\,e^{-ika(\sin\alpha-\sin\varphi)}\Big]\frac{e^{i\left(kr+\frac{\pi}{4}\right)}}{\sqrt{2\pi kr}}, \\
E_\varphi = H_z = H_{0z}\cdot\Big[g^0(1)\,e^{ika(\sin\alpha-\sin\varphi)} + {} \\
+ g^0(2)\,e^{-ika(\sin\alpha-\sin\varphi)}\Big]\frac{e^{i\left(kr+\frac{\pi}{4}\right)}}{\sqrt{2\pi kr}}.
\end{aligned}
\right\} \tag{6.08}
$$

Here the first terms correspond to the waves from edge 1 (y = a), and the second terms correspond to the waves from edge 2 (y = -a). The functions f^0 and g^0 are determined in the right half-plane $\left(|\varphi|<\frac{\pi}{2}\right)$ by the equations

$$
\left.
\begin{aligned}
f^0(1) = -f^0(2) = \frac{\cos\alpha}{\sin\alpha-\sin\varphi}, \\
g^0(1) = -g^0(2) = \frac{\cos\varphi}{\sin\alpha-\sin\varphi}.
\end{aligned}
\right\} \tag{6.09}
$$

Now let us find the field radiated by the nonuniform part of the current. Assuming the strip is sufficiently wide (ka >> 1), one is able to approximately consider that the current near its upper edge is the same as on the ideally conducting half-plane $-\infty\leqslant y\leqslant a$, and near the lower edge it is the same as on the half-plane $-a\leqslant y\leqslant\infty$. Therefore, in accordance with § 4, the field from the nonuniform part of the current flowing on the strip may be represented in the form of the sum of the edge cylindrical waves.

$$
\left.
\begin{aligned}
E_z = -H_\varphi = E_{0z}\cdot\Big[f^1(1)\,e^{ika(\sin\alpha-\sin\varphi)} + {} \\
+ f^1(2)\,e^{-ika(\sin\alpha-\sin\varphi)}\Big]\frac{e^{i\left(kr+\frac{\pi}{4}\right)}}{\sqrt{2\pi kr}}, \\
E_\varphi = H_z = H_{0z}\cdot\Big[g^1(1)\,e^{ika(\sin\alpha-\sin\varphi)} + {} \\
+ g^1(2)\,e^{-ika(\sin\alpha-\sin\varphi)}\Big]\frac{e^{i\left(kr+\frac{\pi}{4}\right)}}{\sqrt{2\pi kr}},
\end{aligned}
\right\} \tag{6.10}
$$

where the functions f^1 and g^1 are determined in the right half-plane $\left(|\varphi|<\frac{\pi}{2}\right)$ by the equations

$$\left.\begin{aligned} f^1(1)=f(1)-f^0(1), \quad f^1(2)=f(2)-f^0(2), \\ g^1(1)=g(1)-g^0(1), \quad g^1(2)=g(2)-g^0(2), \end{aligned}\right\} \tag{6.11}$$

in connection with which

$$\left.\begin{aligned} f(1)&=\frac{\cos\frac{\alpha+\varphi}{2}-\sin\frac{\alpha-\varphi}{2}}{\sin\alpha-\sin\varphi}, \\ f(2)&=-\frac{\cos\frac{\alpha+\varphi}{2}+\sin\frac{\alpha-\varphi}{2}}{\sin\alpha-\sin\varphi}, \\ g(1)&=\frac{\cos\frac{\alpha+\varphi}{2}+\sin\frac{\alpha-\varphi}{2}}{\sin\alpha-\sin\varphi}, \\ g(2)&=\frac{-\cos\frac{\alpha+\varphi}{2}+\sin\frac{\alpha-\varphi}{2}}{\sin\alpha-\sin\varphi}. \end{aligned}\right| \tag{6.12}$$

The functions f^0 and g^0 are described by the relationships (6.09).

As a result, the fringing field (the sum of the fields radiated by the uniform and nonuniform parts of the current) will equal

$$\left.\begin{aligned} E_z=-H_\varphi=E_{0z}\cdot[f(1)\,e^{ika(\sin\alpha-\sin\varphi)}+ \\ +f(2)\,e^{-ika(\sin\alpha-\sin\varphi)}]\frac{e^{i\left(kr+\frac{\pi}{4}\right)}}{\sqrt{2\pi kr}}, \\ E_\varphi=H_z=H_{0z}\cdot[g(1)\,e^{ika(\sin\alpha-\sin\varphi)}+ \\ +g(2)\,e^{-ika(\sin\alpha-\sin\varphi)}]\frac{e^{i\left(kr+\frac{\pi}{4}\right)}}{\sqrt{2\pi kr}}. \end{aligned}\right| \tag{6.13}$$

Consequently, the resulting field is expressed only in terms of the functions f and g which determine the cylindrical wave in the rigorous solution (see § 2). The field is the superposition of two such waves which diverge from the edges 1 ($y = a$) and 2 ($y = -a$).

Substituting into Equations (6.13) the explicit Expressions (6.12) for the functions f and g, we obtain

$$E_z = -H_\varphi = E_{0z}\left\{-\frac{\cos[ka(\sin\alpha - \sin\varphi)]}{\cos\frac{\alpha+\varphi}{2}} + \right.$$
$$\left. + i\,\frac{\sin[ka(\sin\alpha - \sin\varphi)]}{\sin\frac{\alpha-\varphi}{2}}\right\}\frac{e^{i\left(kr+\frac{\pi}{4}\right)}}{\sqrt{2\pi kr}},$$
$$E_\varphi = H_z = H_{0z}\left\{\frac{\cos[ka(\sin\alpha - \sin\varphi)]}{\cos\frac{\alpha+\varphi}{2}} + \right.$$
$$\left. + i\,\frac{\sin[ka(\sin\alpha - \sin\varphi)]}{\sin\frac{\alpha-\varphi}{2}}\right\}\frac{e^{i\left(kr+\frac{\pi}{4}\right)}}{\sqrt{2\pi kr}}. \qquad (6.14)$$

These equations are valid when $r \gg ka^2$ and $|\varphi| < \frac{\pi}{2}$. Moreover, it is assumed that $ka \gg 1$, since only under this condition is one able to consider the nonuniform part of the current in the vicinity of the strip's edge to be approximately the same as on the corresponding half-plane. In the case of normal incidence of a plane wave ($\alpha = 0$), Equations (6.14) change into expressions corresponding to the first approximation of Schwarzschild [15].

From relationships (6.03) and (6.05), it follows that the electric field is an even function, and the magnetic field an odd function, of the x coordinate measured perpendicular to the plane $x = 0$ (in which the current flows)

$$E_z(x) = E_z(-x), \quad H_z(x) = -H_z(-x). \qquad (6.15)$$

Therefore, on the basis of Equations (6.14) and (6.15) one is able to write the expressions for the fringing field in the region $x < 0$ (where $\frac{\pi}{2} < |\varphi| < \pi$)

$$E_z = -H_\varphi = \pm E_{0z}\left\{\frac{\cos[ka(\sin\alpha - \sin\varphi)]}{\sin\frac{\alpha-\varphi}{2}} - \right.$$
$$\left. - i\,\frac{\sin[ka(\sin\alpha - \sin\varphi)]}{\cos\frac{\alpha+\varphi}{2}}\right\}\frac{e^{i\left(kr+\frac{\pi}{4}\right)}}{\sqrt{2\pi kr}},$$
$$E_\varphi = H_z = \pm H_{0z}\cdot\left\{\frac{\cos[ka(\sin\alpha - \sin\varphi)]}{\sin\frac{\alpha-\varphi}{2}} + \right. \qquad (6.16)$$

$$+i\frac{\sin[ka(\sin\alpha-\sin\varphi)]}{\cos\frac{\alpha+\varphi}{2}}\Bigg\}\frac{e^{i\left(kr+\frac{\pi}{4}\right)}}{\sqrt{2\pi kr}}. \tag{6.16}$$

Here one must select the upper sign in front of the braces when $\phi > 0$, and one must select the lower sign when $\phi < 0$.

The resulting Equations (6.14) and (6.16), in contrast to Equations (6.07), satisfy the reciprocity principle. It is not difficult to establish this by verifying that Equation (6.14) is not changed with the simultaneous replacement of α by ϕ and of ϕ by α, and Equation (6.16) is not changed with the replacement of α by $\pi + \phi$ and of ϕ by $\alpha - \pi$ (if $-\pi<\varphi<-\frac{\pi}{2}$) and with the replacement of α by $\pi - \phi$ and of ϕ by $\pi - \alpha$ (if $\frac{\pi}{2}<\varphi<\pi$).

However, the indicated equations lead to a discontinuity of the magnetic vector tangential component H_z on the plane $x = 0$. This is connected with the fact that, by considering the nonuniform part of the current in the vicinity of the strip's edge to be the same as on the corresponding half-plane, we actually assume the presence of currents on the entire plane containing the strip. In order to refine the resulting expressions, it is necessary to solve the problem of secondary diffraction — that is, diffraction of the wave travelling from one edge of the strip to its other edge. In other words, it is necessary to take into account the diffraction interaction of the strip's edges. As we see, it is also necessary to take into account the secondary diffraction in the case $\alpha=\pm\frac{\pi}{2}$ when the H_z component of the fringing field must equal zero.

In Chapter V, we will return to the problem of diffraction by a strip, and together with the investigation of the secondary diffraction, we will present the results of the numerical calculation based on Equations (6.07), (6.14) and (6.16).

FOOTNOTE

Footnote (1) on page 27

The designations used here differ slightly from those used in the papers [7 - 11]. The functions f and f^1 there were designated by f_1 and f, respectively.